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$$(E-H)P\Psi = 0$$

$$H = \mathcal{H}_{PP} + \mathcal{H}_{PQ} \frac{1}{E - \mathcal{H}_{QQ}} \mathcal{H}_{QP}$$

$$(\mathcal{E}_s - \mathcal{H}_{QQ})\Phi_s = 0$$

$$\rho_s = \Phi_s \rangle\langle \Phi_s$$

$$H = H' + \mathcal{H}_{PQ} \frac{\rho_s}{E - \mathcal{E}_s} \mathcal{H}_{QP}$$

$$T(\beta|\alpha) = T_P(\beta|\alpha) + T_R(\beta|\alpha)$$

$$T_R(\beta|\alpha) = \frac{\langle \psi_\beta^- \mathcal{H}_{PQ} \Phi_s \rangle\langle \Phi_s \mathcal{H}_{QP} \psi_\alpha^+ \rangle}{E - \mathcal{E}_s + i\pi \langle \Phi_s \mathcal{H}_{QP} \delta(E-H') \mathcal{H}_{PQ} \Phi_s \rangle}$$

$$H'\psi = E\psi$$

$$P = P_0^{(N)} p_0^{(\gamma)} + P_1^{(N)} p_1^{(\gamma)} \equiv P_0 + P_1$$

$$p_0^{(\gamma)} p_1^{(\gamma)} = 0$$

$$Q = 1 - P = 1 - P_0^{(N)} p_0^{(\gamma)} - P_1^{(N)} p_1^{(\gamma)} \simeq Q_0^{(N)} p_0^{(\gamma)} + Q_1^{(N)} p_1^{(\gamma)} =$$

$$= Q_0 + Q_1$$

$$H' = P_0 H' P_0 + P_1 H' P_1 + P_0 H' P_1 + P_1 H' P_0$$

$$(E - H')[\psi(0) + \psi(1)] = 0.$$

$$\cancel{(E - P_0 H' P_0 - P_0 H' P_1)[\psi(0) + \psi(1)]} = 0$$

$$\cancel{E - P_0 H' P_0}$$

$$(E - P_0 H' P_0 - P_1 H' P_0)\,\psi(0) + (E - P_1 H' P_1 - P_0 H' P_1)\,\psi(1) = 0$$

Mult. por $p_0^{(\gamma)}$

$$(E - P_0 H' P_0)\,\psi(0) - P_0 H' P_1\,\psi(1) = 0$$

$$(E - H'_N)\,\psi(0) = (P_0 H' P_1)\,\psi(1)$$

Mult. por $\gamma_1^{(\gamma)}$

$$(E - P_1 H' P_1)\,\psi(1) - P_1 H' P_0\,\psi(0) = 0$$

$$\therefore (E - \hbar\omega - H'_N)\,\psi(1) = P_1 H' P_0\,\psi(0)$$

$$\boxed{\begin{aligned} (E - H'_N)\,\psi(0) &= (P_0 H' P_1)\,\psi(1) \\ (E - \hbar\omega - H'_N)\,\psi(1) &= (P_1 H' P_0)\,\psi(0) \end{aligned}}$$

$$T = \cancel{\langle \psi^-(1) \left[H'_N + P_0 H' P_1 \frac{1}{E - \hbar\omega - H'_N} P_1 H' P_0 - E \right]} \Phi \rangle$$

$$T_p(\beta|\alpha) = \langle \psi_{\beta 1}^-(1)\, P_1 H' P_0\, \psi_{\alpha 0}^+(0) \rangle$$

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$$\mathcal{H}_{QQ} = Q_0 \mathcal{H}'_{QQ} Q_0 + Q_1 \mathcal{H}_{QQ} Q_1 + Q_0 \mathcal{H}_{QQ} Q_1 + Q_1 \mathcal{H}_{QQ} Q_0.$$

$$(\varepsilon_s - \mathcal{H}_{QQ})[\Phi_s(0) + \Phi_s(1)] = 0$$

$$(\varepsilon_s - Q_0 \mathcal{H}_{QQ} Q_0 - Q_1 \mathcal{H}_{QQ} Q_0)\,\Phi_s(0) + (\varepsilon_s - Q_1 \mathcal{H}_{QQ} Q_1 - Q_0 \mathcal{H}_{QQ} Q_1)\,\Phi_s(1) = 0$$

$$(\varepsilon_s - H_{QQ}^{(N)})\,\Phi_s(0) = H_{QQ}^{(\gamma N)}\,\Phi_s(1)$$

$$(\varepsilon_s - \hbar\omega - H_{QQ}^{(N)})\,\Phi_s(1) = H_{QQ}^{(\gamma N)}\,\Phi_s(0)$$

$$\langle \Phi_s H_{QP} \psi_\alpha^+ \rangle = \langle [\Phi_s(0) + \Phi_s(1)]\, H_{QP} [\psi_\alpha^+(0) + \psi_\alpha^+(1)] \rangle =$$

$$= \langle \Phi_s(0)\, H_{QP}^N \psi_\alpha^+(0) \rangle + \langle \Phi_s(0)\, H_{QP}^{\gamma N} \psi_\alpha^+(1) \rangle +$$

$$+ \langle \Phi_s(1)\, H_{QP}^N \psi_\alpha^+(1) \rangle + \langle \Phi_s(1)\, H_{QP}^{\gamma N} \psi_\alpha^+(0) \rangle$$

$$\langle \psi_{\beta 1}^-(0)\, H_{PQ}^N\, \Phi_{s0}(0) \rangle = \langle (E - P_0 H' P_1)\, \psi_{\beta 1}^-(1)\, \Phi_{s0}(0) \rangle$$

$$(E - \hbar\omega - H_{QQ}^N)\,\Phi_\beta^{(-)} = [E - \hbar\omega - H_{QQ}^N + H_{QP}]\,\psi_\beta^{(-)}$$

$$\cancel{\Phi_\beta^{(-)} = \psi_\beta^- + Q\,\Phi_\beta^{(-)}}$$

$$\cancel{(E - \hbar\omega - H_{QQ}^N)\,\psi_\beta^- + (E - \hbar\omega - H_{QQ}^N)\,Q\,\Phi_\beta^{(-)}} =$$

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$$(E - H_{QQ})\, Q\Phi = H_{QP}\,\psi$$

$$(E - \cancel{H_{PP}} - H_{QQ})(\Phi - \psi) = H_{QP}\,\psi$$

$$[E - Q(H^{N} + H^{\gamma} + H^{\gamma N})Q]\,\Phi = (E - H_{QQ} + H_{QP})\psi$$

$$[E - H^{N}_{QQ} - \hbar\omega]\,\Phi_{\beta}^{(-)} = (E - \hbar\omega - H^{N}_{QQ} + H_{QP})\psi$$

$$\left[H_{QP}^{\gamma N} + H_{QQ}^{\gamma N}\frac{1}{E - \hbar\omega - H^{N}_{QQ}}H^{N}_{QP}\right]\psi_{\beta}^{-}(1) = \qquad Q(P\Phi + Q\Phi)$$

$$\left[H_{QP}^{\gamma N} + H_{QQ}^{\gamma N}\frac{1}{E - \hbar\omega - H^{N}_{QQ}}(H_{QP} - H_{QP}^{\gamma N})\right]\psi_{P}^{-}(1)\Big] =$$

$$\left[H_{QP}^{\gamma N} + H_{QQ}^{\gamma N}(\Phi_{\beta}^{(-)} - \psi_{\beta}^{(-)}) - H_{QQ}^{\gamma N}\frac{1}{E - \hbar\omega - H^{N}_{QQ}}H_{QP}^{\gamma N}\psi_{\beta}^{-}(1)\right] =$$

$$Q[H^{\gamma N}P + H^{\gamma N}Q]\,\Phi_{\beta}$$

$$= \left[Q H^{\gamma N}\Phi_{\beta} - H_{QQ}^{\gamma N}\left(1 + \frac{1}{E - \hbar\omega - H^{N}_{QQ}}H_{QP}^{\gamma N}\right)\Phi_{\beta}^{-}(1)\right] =$$

$$Q\, H^{\gamma N}\Phi$$

$$= (E - \hbar\omega - H^{N}_{QQ})\,\Phi_{\beta} - (E - \hbar\omega - H^{N}_{QQ})\,\psi_{\beta} = H_{QP}\,\psi_{\beta}$$

$$(E - \hbar\omega - H^{N}_{QQ})\,Q\Phi = H_{QP}\,\psi_{\beta} \longrightarrow Q H^{\gamma N}P\psi + Q H^{\gamma N}Q\Phi$$

$$H_{QP}^{\gamma N}\psi_{\beta} + H_{QQ}^{\gamma N}(\Phi_{\beta}^{(-)} - \cancel{\psi_{\beta}}) = H_{QQ}^{\gamma N}\Phi_{\beta}^{-} + (H_{QP}^{\gamma N} - H_{QQ}^{\gamma N})\psi_{\beta}$$

$$= H_{QQ}^{\gamma N}\Phi + Q \qquad P\psi + Q\Phi$$

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find a very close agreement between the resulting electron energies and the expected appearance potentials of O^- and C^-. In Table I we have listed these energies and the appearance potentials for negative ions derived from CO if the dissociation energy of CO is taken as 11.11 ev and the affinity of O as 1.45 ev.[4] For $C^- + O^+$ we have listed values taken from the theses of Lagegren[5] and Petrocelli[6] separately. In CO, as in H_2, we find at least one very broad peak below the first one that appears from free electron capture. In the present case the ion formed would have to be CO^- if our ideas are correct. A mass spectrographic analysis is under way to elucidate these processes further.

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[1] R. Curran and T. M. Donahue (to be published); Bull. Am. Phys. Soc. 4, 234 (1959).
[2] G. J. Schulz, Phys. Rev. 113, 846 (1959).
[3] V. I. Khvostenko and V. M. Dukel'skii, J. Exptl. Theoret. Phys. U.S.S.R. [translation: Soviet Phys. JETP 6, 457 (1958)].
[4] L. M. Branscomb and S. J. Smith, Phys. Rev. 98, 1127 (1955).
[5] C. R. Lagegren, thesis, University of Minnesota, 1956, Dissertation Abstr. 16, 770 (1956).
[6] A. W. Petrocelli, thesis, Providence University, 1958 (unpublished).

DIPOLE STATE IN NUCLEI*

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(Received October 28, 1959)

In order to explain the unexpectedly high (γ, p) cross section in heavy nuclei, various authors[1,2] have proposed that these protons arise mainly from a direct process. The close relationship of this process to the shell model and optical model has been elucidated by Wilkinson,[3] who points out that the initial state of the nucleus is quite well described by the shell model. The proton involved in the direct process can then be considered as being initially in an eigenstate in the shell-model well. Upon absorption of the dipole gamma ray, the proton makes a transition to either a bound level in the well or one in the continuum. Because this state is not stationary, it is given a width $\Gamma + 2W$, where W is the absorption in the optical-model well at the relevant excitation and describes the absorption of the single-particle excitation into compound states and Γ is the width for escape. The proportion of fast protons that escapes is then $\Gamma/(\Gamma + 2W)$. The picture is very appealing, in that it produces the observed order of magnitude of fast particles, which is several orders of magnitude greater than the statistical description predicts. The relation of this description to one in terms of compound states of the system has been given in detail.[4] In reference 4 it is made clear that the highly excited levels discussed by Wilkinson are really combinations of thousands or millions of compound states which, however, act coherently as a single-particle state for some processes. We will refer to these groups of compound states as single-particle excitations.

The positions of the single-particle excitations can be found directly through other processes. Recently, (d, p) experiments using poor resolution[5,6] have determined the positions of single-particle excitations lying between zero bombarding energy and the binding energy of the last neutron. The spacings of these excitations seem to be in sharp conflict with those required by Wilkinson. For example, in Ti^{49}, the $f_{7/2}$ and $g_{9/2}$ levels are only about 4 Mev apart. However, just the transition between these two levels is an appreciable part of the giant dipole resonance in this nucleus, which comes at an energy of about 15 Mev. It is true, of course, that one must add a pairing energy to the 4 Mev before making the comparison, because in the absorption of the gamma ray a pair is generally broken. However, this is only one or two Mev. It seems, therefore, that the transition between single-particle excitations should occur at an energy of only about half that of the giant dipole resonance.

We should like to point out in this note that these two energies cannot be compared directly, since, in the dipole absorption, a hole is formed in the nucleus. Since the process is a dipole one, the excited particle and hole are strongly correlated in angle; i.e., their angular momentum must be coupled to form a 1- state, assuming the original nucleus to be in a 0+ state. Because many particle-hole states can be formed, and because these states are almost degenerate in energy, the particle-hole interaction can have a profound effect in redistributing dipole transition strength.

We shall demonstrate these effects by using a schematic model, the mathematics of which is suggested by the Copenhagen work on pairing interactions.[7] Major numerical approximations are made in going from the actual situation to this rough schematic model. The model does, however, exhibit how coherent effects are able to push the dipole transitions to much higher energies than one would, at first sight, think possible.

We consider first protons in a potential well, and will indicate the extension to the case of protons and neutrons in a nucleus later. The main contributions to the absorption come from closed shells. We shall, therefore, specialize our discussion to nuclei with double closed shells, neglecting the influence of the few valence nucleons. We neglect spin, and consider only transitions from l to $l+1$. Choosing our "vacuum" as the initial nucleus, we see that the gamma ray creates a particle-hole pair through the process shown in Fig. 1. With axes oriented so that the Hamiltonian describing the interaction with radiation is

$$H_I = e(2\pi\hbar\omega)Z, \tag{1}$$

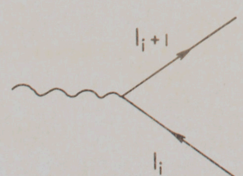
the particle-hole state formed in absorption of the gamma ray is

$$\varphi_i(\vec{r}_p, \vec{r}_h) = (-)^{l_i}[Y_{l_i}(\theta_h, \varphi_h)Y_{l_i+1}(\theta_p, \varphi_p)]_0^1 R_{l_i}(r_h)R_{l_i+1}(r_p), \tag{2}$$

here

$$[Y_{l_i}Y_{l_i+1}]_0^1 = \sum_m C(l_i, l_i+1, 1; m, -m, 0)Y_{l_i}^{\ m}Y_{l_i+1}^{\ -m}.$$

he lower suffixes p and h refer to particle and le, the R's are radial wave functions, and the s are Clebsch-Gordan coefficients. The phase tor $(-1)^{l_i}$ is put in for convenience to make all antities positive. In medium and heavy nuclei, arge number of particle-hole states can be med. The particle-hole interaction will mix se, and to find the perturbed eigenstates, we st solve the secular equation. In realistic uations, this is quite complicated, e.g., see iott and Flowers[8] where this is carried out O^{16}. We shall, therefore, make several



1. Diagram representing the process by a gamma ray creates a particle-hole pair.

approximations which give us a schematic model where we can display the qualitative features explicitly. We shall return later to a discussion of the approximations. First, we shall use zero-range forces, i.e., we take the particle-hole interaction to be

$$V(\vec{r}_p - \vec{r}_h) = V_0\delta(\vec{r}_p - \vec{r}_h),$$

with V_0 positive. (If the particle-particle force is attractive, the particle-hole one is repulsive.) Now when Y_{l_i} and Y_{l_i+1} have the same argument,

$$[Y_{l_i}Y_{l_i+1}]_0^1 = (-)^{l_i}[(l_i+1)/4\pi]^{1/2}Y_1^{\ 0}, \tag{3}$$

so that the diagonal elements of our secular matrix are

$$\epsilon_i + (l_i+1)(V_0/4\pi)\int_0^\infty R_{l_i}^{\ 2}R_{l_i+1}^{\ 2}r^2dr,$$

where the ϵ_i are the unperturbed energies, i.e., the energies of the dipole excitations Wilkinson is considering. The off-diagonal elements are

$$(l_i+1)^{1/2}(l_j+1)^{1/2}(V_0/4\pi)\int_0^\infty R_{l_i}R_{l_i+1}R_{l_j}R_{l_j+1}r^2dr.$$

We next make the further approximation of set-

ting the radial integrals equal, i.e., we set

$$(V_0/4\pi)\int_0^\infty R_{l_i}^{\ 2}R_{l_i+1}^{\ 2}r^2dr = (V_0/4\pi)\int_0^\infty R_{l_i}R_{l_i+1}R_{l_j}R_{l_j+1}r^2dr = G, \tag{4}$$

giving us a secular equation

$$0 = \begin{vmatrix} \epsilon_1 + (l_1+1)G - \lambda & (l_1+1)^{1/2}(l_2+1)^{1/2}G & (l_1+1)^{1/2}(l_3+1)^{1/2}G & \cdots \\ (l_1+1)^{1/2}(l_2+1)^{1/2}G & \epsilon_2 + (l_2+1)G - \lambda & (l_2+1)^{1/2}(l_3+1)^{1/2}G & \cdots \\ (l_1+1)^{1/2}(l_3+1)^{1/2}G & (l_2+1)^{1/2}(l_3+1)^{1/2}G & \epsilon_3 + (l_3+1)G - \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{vmatrix}$$

By collecting powers of $(\epsilon_i - \lambda)$ one can easily show that this is equivalent to the equation

$$\prod_{i=1}^{n}(\epsilon_i - \lambda) + \sum_{j=1}^{n}\prod_{i=1(i\neq j)}^{n}(\epsilon_i - \lambda)(l_j+1)G = 0, \tag{6}$$

which in turn is equivalent to

$$\sum_{j=1}^{n}(l_j+1)/(\lambda - \epsilon_j) = 1/G. \tag{7}$$

The solutions of Eq. (7) can be obtained by plotting the left- and right-hand sides graphically as in Fig. 2, where the ×'s show the solutions λ_n. It is seen that the uppermost eigenvalue is pushed up a large amount. Let us now ask what happens when the ϵ_i become degenerate. Then, it easily is found that the solutions λ_i are

$$\lambda_1 = \lambda_2 = \cdots = \lambda_{n-1} = \epsilon, \qquad \lambda_n = \epsilon + \sum_{i=1}^{n}(l_i+1)G, \tag{8}$$

where ϵ is the common value of the ϵ_i. Denoting

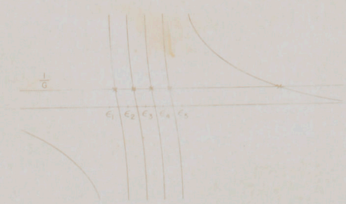


FIG. 2. Graphical solution of Eq. (7).

the perturbed eigenstates by χ_i, the nth state is

$$\chi_n = [\sum(l_i+1)]^{-1/2}\sum(l_j+1)^{1/2}\varphi_j. \tag{9}$$

Therefore, this state is pushed up through coherent effects from all of the degenerate levels. Since a large number of particle-hole states participate and since each term in the sum in Eq. (8) is of the order of one Mev, the nth level is pushed up by several Mev. Now, the dipole transition amplitude to any one of the unperturbed states φ_i is just

$$T_i = \int\varphi_i(\vec{r}_p, \vec{r}_h)\delta(\vec{r}_p - \vec{r}_h)H_I(\vec{r}_p)d^3r_p d^3r_h$$

$$= [(l_i+1)/3]^{1/2}\int_0^\infty R_{l_i}R_{l_i+1}r^3dr. \tag{10}$$

Assuming, again, that the radial integrals are all equal, it is clear from Eq. (9) and the orthogonality of the φ_i that the uppermost level carries, in this approximation, all of the dipole transition strength. That is, through the particle-hole interaction, the lower levels have been denuded of their dipole transition strength and this has been transferred into the uppermost level. We propose, therefore, to call this level the "dipole level."

These arguments can be generalized to cover the transitions l_i to l_i-1, as well, by changing l_i to l_i-1 in the relevant matrix elements of the secular matrix and in T_i. However, whereas the approximation of setting all radial integrals equal does not seem unreasonable in the transitions l_i to l_i+1 where the important transitions are "nodeless to nodeless" ones, it would not seem to be justified in the transitions l_i to $l_i - 1$. However, in heavy nuclei (e.g., see Wilkinson's calculations for Sn and Pb in reference 3) almost all of the oscillator strength is contained in the transitions $l_i + 1$.

To take account of effects due to the neutrons in the nucleus, we consider first the case of light nuclei and neglect Coulomb forces, so that isotopic spin can be considered to be a good quantum number. Then the dipole interaction Hamiltonian, after removal of center-of-mass coordinates, is

$$H_I = (e/2)(2\pi\hbar\omega)Z\tau_3, \tag{11}$$

for a nucleus with equal number of protons and neutrons; i.e., the effective charge on the proton is $e/2$, that on the neutron, $-e/2$. Since the Hamiltonian is the third component of a vector in isotopic spin space, only $T=1$ states will be formed from applying it to the $T=0$ ground state. The particle-hole states will now be

$$(2)^{-1/2}[\varphi_i^{+}(\vec{r}_p, \vec{r}_h) - \varphi_i^{-}(\vec{r}_p, \vec{r}_h)], \tag{12}$$

where the + indicates that the first part of the wave function refers to a proton particle-hole state, and the - indicates that the other part refers to a neutron particle-hole state. In the $T=0$ state, which cannot be formed by absorption of the gamma ray, the - sign would be replaced by a + sign.

Now the question of what happens to the $T=0$ and $T=1$ levels depends on the isotopic spin dependence of the force. A repulsive particle-hole interaction containing no isotopic spin dependence will push the $T=0$ level up in energy, leaving the $T=1$ level unchanged, whereas a force of character $\vec{\tau}_i \cdot \vec{\tau}_j$ which is repulsive for like particles will push the $T=1$ level up. The Rosenfeld mixture used by Elliott and Flowers[8] is of the latter type. A more detailed discussion of effects of a force such as the Rosenfeld mixture would require introduction of spin, but there is no doubt that the isotopic spin dependence is such as to push the $T=1$ levels up.

In heavy nuclei, where isotopic spin is no longer a good quantum number, neutron and proton excitations can be treated independently. Whereas it is true that the neutron particle-hole states are created with opposite phase from the proton particle-hole ones because of the τ_3 in the interaction Hamiltonian, a force of the $\vec{\tau}_i \cdot \vec{\tau}_j$ character will give opposite signs when evaluated between particle-hole states of like particles and particle-hole states of unlike particles so that the matrix elements in the secular matrix will again tend to be all positive.

We have made radical approximations in this schematic model, but the qualitative features should be given correctly, at least for heavy nuclei. Unfortunately, detailed calculations without our simplifying assumptions are difficult to make here, and these have been carried out only for the case of O^{16}. Here, the work of Elliott and Flowers does show that the dipole transitions occur at a high energy as a result of the particle-hole interactions, but the dipole strength is large not only in the top level, but in both of the two highest levels, although the lower levels are denuded as we would predict. Nothing in our model would predict that two levels should be pushed up.

In this case of a light nucleus, there are, however, several features which are not at all well described in our picture, and may be responsible for this. First, spin-flip transitions are relatively important here, because of the low l involved, and the unperturbed energies ϵ_i for these transitions are quite different from those of the non-spin-flip transitions. (We have neglected spin in our model, but it is clear that when generalized to include spin, it will describe only the non-spin-flip transitions well, because only these are nearly degenerate in energy.) Furthermore, the transitions l_i to $l_i - 1$ are relatively important in oxygen. Neither of these features is present in the medium and heavy weight nuclei, where almost all of the oscillator strength is carried by the l_i to $l_i + 1$ transitions involving no spin flip.

Some general qualitative features come out of the work of Elliott and Flowers which are present in our model. In particular, it is true that the state which is pushed highest up in energy is the most symmetric state, in terms of the particle and hole, i.e., that is the state in which expansion of the χ_i in terms of the φ_i such as in Eq. (10) involves mostly + signs with our choice of phases for the φ_i. Since the matrix elements of the secular matrix are all positive, clearly this state will lie highest in energy. This is analogous to the most symmetric state lying lowest in energy when attractive forces are present, as was noted long ago by Wigner and collaborators.[9] The more symmetric the state is, the more dipole strength it carries.

We believe that our schematic model indicates

that an increasing regularity will occur in going towards heavier nuclei, so that the coherent effects can become strong enough to shift the dipole transitions up several Mev. The schematic model is, of course, no substitute for detailed calculations, but indicates the possibility of these coherent effects in a simple way.

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[1] P. Jensen, Naturwiss. 35, 190 (1948).

[2] E. D. Courant, Phys. Rev. 82, 703 (1951).

[3] D. H. Wilkinson, Physica 22, 1039 (1956).

[4] G. E. Brown and J. S. Levinger, Proc. Phys. Soc. (London) A71, 733 (1958).

[5] Schiffer, Lee, and Zeidman (to be published).

[6] Cohen, Mead, Price, Quisenberry, and Martz (to be published).

[7] See, especially, the so-called "degenerate model" for pairing interactions, developed mainly by B. Mottelson, which will appear in an account of the Copenhagen work.

[8] J. P. Elliott and B. F. Flowers, Proc. Roy. Soc. (London) A242, 57 (1957).

[9] This was pointed out to one of the authors by Professor G. Breit.

Dispersion Formalism for Radiative Capture

Lane and Lynn (Nucl. Phys $\underline{17}$, (1960), 563)

Incident particles α', spin $i_{\alpha'}$ } relative ang. momentum l'.
Target nuclei β', spin $i_{\beta'}$

Channel spin: $\underline{s}' = \underline{i}_{\alpha'} + \underline{i}_{\beta'}$

Total spin $\underline{J} = \underline{s}' + \underline{l}'$

Total wave-function Ψ_{EJM} (E = total energy), in the channel $c' \equiv (\alpha', \beta', s', l')$

$$\Psi_{EJM} = v'^{\frac{1}{2}} \left[I_{c'}(k'r) - U^J_{c',c'}\, O_{c'}(k'r) \right] \varphi_{c'JM}$$

where: $\varphi_{c'JM}$ is the channel function (intrinsic wave-function of α' and β' coupled to channel spin s' and to the ang. functions $i^{l'} Y_{l'm}/r$)

v' is the velocity in the channel and is related to the bombarding energy ε' by:

$$\varepsilon' = \frac{1}{2} M' v'^2 = \frac{\hbar^2 k'^2}{2M'}$$

where M' is the reduced mass. $I_{c'}$ and $O_{c'}$ are the spherical Bessel functions with asymptotic form:

$$I_{c'}(k'r) \to e^{-i(k'r - \frac{1}{2}l'\pi)} \qquad O_{c'}(k'r) \to e^{i(k'r - \frac{1}{2}l'\pi)}$$

$U^J_{c'c'}$ is the diagonal element of the scattering matrix.

The cross-section for radiative capture of α' by β' to form the final bound state $\Psi_{fJ_fM_f}$ has the form:

$$\sigma_{\alpha'\beta',\gamma f} = \frac{\pi}{k'^2} \sum_J \frac{2J+1}{(2i_{\alpha'}+1)(2i_{\beta'}+1)} \sum_{s'l'} \left| U^J_{\gamma f, c'} \right|^2$$

where $U^J_{\gamma f, c'}$ is the scattering matrix element given by:

$$U^J_{\gamma f, c'} = \left(\frac{16\pi}{9\hbar} \right)^{\frac{1}{2}} k_\gamma^{\frac{3}{2}} \frac{\langle \Psi_{fJ_f} \| \mathcal{H}^{(1)}_q \| \Psi_{EJ} \rangle}{(2J+1)^{1/2}}$$

The reduced matrix element is:

$$\left(\frac{2J_f+1}{2J+1} \right)^{\frac{1}{2}} \sum_{M+q=M_f} \langle JM1q | J_fM_f \rangle \int \Psi^*_{fJ_fM_f} \mathcal{H}^{(1)}_q \Psi_{EJM}$$

where the integration is over all coordinates, the summation is for fixed M_f and $\mathcal{H}^{(1)}_q$ is the dipole operator of comp. q and k_γ the photon wave number corresponding to energy ε_γ.

Let's consider $\underline{\text{only}}$ the hard sphere contribution:

In the channel $c_f = (\alpha' \beta' s_f l_f)$

$$\Psi_{fJ_fM_f} = \left(\frac{2}{R} \right)^{\frac{1}{2}} \theta_{fc_f} \varphi_{c_fJ_fM_f} \frac{O_{c_f}(k_f r)}{O_{c_f}(k_f R)}$$

where

$$\theta_{fc_f} = \gamma_{fc_f} \left(\frac{\hbar^2}{MR^2} \right)^{-\frac{1}{2}}$$

$$\gamma_{fc_f} = \left(\frac{\hbar^2}{2M'R} \right)^{\frac{1}{2}} \int X_{fJM} \varphi^*_{c_fJM}\, dS$$

$$\therefore U^J_{\gamma_f, c_f} = U = \left(\frac{16\pi}{9\hbar v'} \right)^{\frac{1}{2}} k_\gamma^{\frac{3}{2}} \sum_{c_f} \theta_{fc_f} \frac{\langle r\varphi_{c_f} \| H^{(1)}_q \| r\varphi_{c'J} \rangle}{(2J+1)^{1/2}}\, 2ik' R^{\frac{5}{2}} e\, I_{l'l_f}$$

where:

$$I_{l'l_f} = \frac{\sqrt{2}}{R^3 e} \int \frac{O_{c_f}(k_f r)}{O_{c_f}(k_f R)} h^{(1)} \psi_{HS}(r)\, r\, dr$$

and ψ_{HS} is the hard sphere wave function for incident energy ε'

$$\psi_{HS} = \frac{1}{2k'r} \left(I_{c'} - e^{-2i\phi_{c'}} O_{c'} \right)$$

($\phi_{c'}$ is the hard-sphere scattering phase angle $= -k'R$ for s-wave neutrons)

The dipole operator $\mathcal{H}^{(1)}_q$ has been split into the angle-intrinsic spin part $H^{(1)}_q$ and the radial part $h^{(1)}$:

$$\mathcal{H}^{(1)}_q = h^{(1)} H^{(1)}_q$$

For E1 transitions:

$$h^{(1)}(r) = \bar{e} r \qquad H^{(1)}_q = Y_{1,q}(\Omega)$$

where $\bar{e}$ is the effective charge ($-\frac{Ze}{A}$ for neutrons, $\frac{Ne}{A}$ for protons)

We can show:

$$\frac{\langle r\varphi_{c_f J_f} \| H^{(1)} \| r\varphi_{c' J}\rangle}{(2J+1)^{1/2}} = \left(\frac{3}{8\pi}\right)^{\frac{1}{2}} (-)^{\frac{1}{2}(l'-l_f-1)+J_f-J} \cdot \delta_{s',s_f} \frac{(l'+l_f+1)^{\frac{1}{2}}}{(2l'+1)^{1/2}} U(1 l_f J s', l' J_f)$$

Then, we can find.

$$\sigma_{\alpha'\beta'\gamma f} = \frac{8}{3}\pi e^2 \frac{R^5}{\hbar v'} k_\gamma^3 \sum_{l' l_f s} \frac{(2J_f+1)(l_f+l'+1)}{(2l_f+1)(2i_{\alpha'}+1)(2i_{\beta'}+1)} \theta_{f c_f} |I_{l' l_f}|^2$$

ON THE THEORY OF RADIATIVE PROCESSES IN NUCLEAR REACTIONS

Total system:

$$H = H(1,2,\dots,n, \underline{b}_i, \underline{b}_j^+) = \sum_{i=1}^{n} T_i + \sum_{i<j}^{n} V_{ij} + \sum_k \underline{b}_k^+ \underline{b}_k \omega_k - \frac{e}{mc}\sum_{prot} \underline{p}_i \cdot \underline{A}$$

where $\underline{A} = \sum_k (b_k \underline{A}_k + b_k^+ \underline{A}_k^*)$

Symbolically: $H = H_{nucl} + \mathcal{H} + I$

therefore: $H\Psi = E\Psi \quad , \quad \Psi = \Psi(1,2,\dots,n,\underline{b},\underline{b}^+)$.

Model: $H = H_{n-1}(\xi) + T(r) + \mathcal{H} + V(r,\xi) + I(r,\xi,\underline{b},\underline{b}^+)$.

Using the eigenfunctions:

$$H_{n-1}\psi_i = \epsilon_i \psi_i$$

we'll write:

$$\Psi = \sum_i U_i(r,\underline{b},\underline{b}^+)\psi_i(\xi)$$

and we can get, for the incident channel:

$$(T + \mathcal{H} + \mathcal{V} - E)U_0 = 0$$

where the effective potential $\mathcal{V}$ is:

$$\mathcal{V} = V_{00} + I_{00} + (\underset{\sim}{V_0} + \underset{\sim}{I_0}) \frac{1}{E^+ - \underset{\sim}{H}} (\underset{\sim}{I_0}^+ + \underset{\sim}{V_0}^+).$$

$$V_{ij} = \langle\psi_i, V\psi_j\rangle \qquad I_{ij} = \langle\psi_i, I\psi_j\rangle$$

$$\underset{\sim}{V_0} = (V_{01}, V_{02}, \dots) \qquad \underset{\sim}{I_0} = (I_{01}, I_{02}, \dots)$$

$$\underset{\sim}{H} = \| H_{ij} \| \qquad H_{ij} = T\delta_{ij} + \mathcal{H}\delta_{ij} + \epsilon\delta_{ij} + V_{ij} + I_{ij} \quad \text{for } i,j \neq 0.$$

Introducing eigenfunctions of $\underset{\sim}{H}$:

$$\underset{\sim}{H}\,\underset{\sim}{\Phi}_\varepsilon = \varepsilon\,\underset{\sim}{\Phi}_\varepsilon$$

i.e.

$$(H_c + \mathcal{H} + I - \varepsilon)\,\Phi_\varepsilon = 0.$$

where $\quad H_c = \| h_{ij} \|, \qquad h_{ij} = T\delta_{ij} + \epsilon\delta_{ij} + V_{ij}$ (this is Feshbach's).

Solving:

$$\Phi_\varepsilon = \Phi(0) + \frac{1}{\varepsilon - H_c - \mathcal{H}}\, I\, \Phi_\varepsilon$$

where $\quad (\varepsilon - H_c)\,\Phi(0) = 0 \quad$ and $\quad \mathcal{H}\,\Phi(0) = 0.$

To 1st order:

$$\Phi_\varepsilon = \left(1 + \frac{1}{\varepsilon - H_c - \mathcal{H}}\, I\right)\Phi(0) \equiv \Phi_\varepsilon(0) + \Phi_\varepsilon(1).$$

Now:

$$(V_0 + I_0)\,\frac{1}{E^+ - H}\,(I_0^+ + V_0^+) = \sum\!\!\!\!\!\!\int_\varepsilon (V_0 + I_0)\,\frac{\Phi_\varepsilon\rangle\langle\Phi_\varepsilon}{E^+ - \varepsilon}\,(I_0^+ + V_0^+) \simeq$$

$$\simeq \sum\!\!\!\!\!\!\int_\varepsilon (V_0 + I_0)\,\frac{|\Phi_\varepsilon(0) + \Phi_\varepsilon(1)\rangle\langle\Phi_\varepsilon(0) + \Phi_\varepsilon(1)|}{E^+ - \varepsilon}\,(I_0^+ + V_0^+).$$

To 1st order in I we get:

$$(V_0 + I_0)\,\frac{1}{E^+ - H}\,(I_0^+ + V_0^+) = \sum\!\!\!\!\!\!\int_\varepsilon \frac{1}{E^+ - \varepsilon}\Big[\overset{W_{00}}{V_0\Phi(0)\rangle\langle\Phi(0)V_0^+} + \overset{W_{11}}{V_0\Phi(1)\rangle\langle\Phi(1)V_0^+} +$$

$$+ \overset{W_{01}}{V_0\Phi(0)\rangle\langle\Phi(1)V_0^+} + \overset{W_{10}}{V_0\Phi(1)\rangle\langle\Phi(0)V_0^+} + \overset{(VI)_{00}}{V_0\Phi(0)\rangle\langle\Phi(0)I_0^+} + \overset{(VI)_{11}}{V_0\Phi(1)\rangle\langle\Phi(1)I_0^+} +$$

$$+ \overset{(VI)_{01}}{V_0\Phi(0)\rangle\langle\Phi(1)I_0^+} + \overset{(VI)_{10}}{V_0\Phi(1)\rangle\langle\Phi(0)I_0^+} + \overset{(IV)_{00}}{I_0\Phi(0)\rangle\langle\Phi(0)V_0^+} + \overset{(IV)_{11}}{I_0\Phi(1)\rangle\langle\Phi(1)V_0^+} +$$

$$+ \overset{(IV)_{01}}{I_0\Phi(0)\rangle\langle\Phi(1)V_0^+} + \overset{(IV)_{10}}{I_0\Phi(1)\rangle\langle\Phi(0)V_0^+}\Big] \equiv W_{00} + W_{11} + W_{01} + W_{10} + (VI)_{00} + (VI)_{11}$$

$$+ (VI)_{01} + (VI)_{10} + (IV)_{00} + (IV)_{11} + (IV)_{01} + (IV)_{10} \equiv W_{00} + \mathcal{V}'$$

Therefore:

$$\mathcal{V} = \mathcal{V}_0 + \mathcal{V}_I$$

where: $\mathcal{V}_0 = V_{00} + W_{00}$ which is the Feshbach's generalized optical potential

and $\mathcal{V}_I = I_{00} + \mathcal{V}'$.

Transition matrix.

$$\tilde{\lambda}_{if} \equiv \langle\phi, u|\mathcal{V}|U_0^+\rangle \qquad \text{where: } (T + \mathcal{H} - E)|\phi, u\rangle = 0.$$

Introducing the wave functions.

$$(T + \mathcal{H} + \mathcal{V}_0 - E)\,W_0 = 0$$

we'll write:

$$\tilde{\lambda}_{if} = \langle\phi, u|\mathcal{V}_0|W_0^+\rangle + \langle W_0^-|\mathcal{V}_I|U_0^+\rangle.$$

The first term is essentially the Feshbach's transition matrix for nuclear reactions.
The second term contains all radiative processes involving one photon at most.

Assuming $U_0^+ \simeq W_0^+$ we get the terms: (to 1st order in I):

i) $\langle W_0^-|I_{00}|W_0^+\rangle$ the usual one!

ii)
$$\langle W_0^-|W_{01}|W_0^+\rangle = \sum\!\!\!\!\!\!\int \frac{\langle W_0^- V_0\Phi(0)\rangle\langle\Phi(1)V_0^+ W_0^+\rangle}{E^+ - \varepsilon} =$$

$$= \sum\!\!\!\!\!\!\int \frac{\langle W_0^- V_0\Phi(0)\rangle\langle\Phi(0)\, I\, \frac{1}{\varepsilon - H_c - \mathcal{H}}\, V_0^+ W_0^+\rangle}{E^+ - \varepsilon} =$$

$$= \langle W_0^-\, V_0\, \frac{1}{E^+ - H_c}\, I\, \frac{1}{\varepsilon - H_c - \mathcal{H}}\, V_0^+ W_0^+\rangle$$

iii)
$$\langle W_0^-|W_{10}|W_0^+\rangle = \langle W_0^-\, V_0\, \frac{1}{\varepsilon - H_c - \mathcal{H}}\, I\, \frac{1}{E^+ - \varepsilon}\, V_0^+ W_0^+\rangle$$

iv)
$$\langle W_0^-|(VI)_{00}|W_0^+\rangle = \sum\!\!\!\!\!\!\int \frac{\langle W_0^- V_0\Phi(0)\rangle\langle\Phi(0)I_0^+ W_0^+\rangle}{E^+ - \varepsilon}$$

$$= \langle W_0^-\, V_0\, \frac{1}{E - H_c}\, I_0^+ W_0^+\rangle$$

v)
$$\langle W_0^-|(IV)_{00}|W_0^+\rangle = \sum\!\!\!\!\!\!\int \frac{\langle W_0^- I_0\Phi(0)\rangle\langle\Phi(0)V_0^+ W_0^+\rangle}{E^+ - \varepsilon} =$$

$$= \langle W_0^-\, I_0\, \frac{1}{E - H_c}\, V_0^+ W_0^+\rangle$$

The last two cases are the Brown's semi-direct terms!

Radiative Processes in nuclear reactions

Total Hamiltonian:

$$\mathcal{H} = H_A(\xi) + T(r) + W(\nu) + V(r,\xi) + I(r,\xi,\nu)$$

$$V = \sum_{i=1}^{A} V(\vec{r},\vec{r}_i)$$

Electromagnetic field:

$$W = \sum_k \nu_k \omega_k = \sum_k \underbrace{a_k^+ a_k}_{\text{creation and destruction operators respectively.}} \omega_k$$

$$I = -\frac{e}{mc} \sum_{\text{protons}} \vec{p}_i \cdot \vec{A}(\vec{r}_i)$$

$$\vec{A}(r_i) = \sum_k [a_k \vec{A}_k + a_k^+ \vec{A}_k^*]$$

The wave equation is:

$$\mathcal{H} \Psi = E \Psi \qquad (1)$$

Let's use the eigenfunctions:

$$H_A \psi_i = \epsilon_i \psi_i$$

to expand the wave functions as:

$$\Psi(r,\xi,\nu) = \sum_i u_i(r,\nu) \psi_i(\xi)$$

Inserting in (1) we get:

$$(T + W + \epsilon_i + V_{ii} + I_{ii} - E) u_i = -\sum_{j \neq i} (V_{ij} + I_{ij}) u_j$$

where: $\quad V_{ij} = (\psi_i, V \psi_j) \qquad I_{ij} = (\psi_i, I \psi_j)$

We define:

$$\Phi = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \end{pmatrix}$$

$$\Phi = \left[1 + \sum_{n=1} \left(\frac{1}{\varepsilon - H_c - W} I \right)^n \right] \Phi(0)$$

$$V_0 = (V_{01}, V_{02}, \dots)$$

$$I_0 = (I_{01}, I_{02}, \dots)$$

$$H_{ij} = T\delta_{ij} + W\delta_{ij} + \epsilon_i \delta_{ij} + V_{ij} + I_{ij} \qquad \text{for} \quad i,j \neq 0$$

$$(T + W + V_{00} + I_{00} - E)\, u_0 = -(V_0 + I_0)\,\Phi$$

$$(H - E)\,\Phi = -(V_0^+ + I_0^+)\, u_0$$

$\therefore$ The wave equation is:

$$(T + W + \mathcal{V} - E)\, u_0 = 0$$

where the effective potential is:

$$\mathcal{V} = V_{00} + I_{00} + (V_0 + I_0) \frac{1}{E^+ - H} (I_0^+ + V_0^+)$$

Let's study the eigenfunctions of H:

$$H\,\Phi(\varepsilon) = \varepsilon\,\Phi(\varepsilon)$$

$$\therefore \quad (\varepsilon - H_c - W - I)\,\Phi(\varepsilon) = 0$$

where $\quad (H_c)_{ij} = T\delta_{ij} + \epsilon_i \delta_{ij} + V_{ij}$

The formal solution is:

$$\Phi = \Phi(0) + \frac{1}{\varepsilon - H_c - W} I\,\Phi$$

where $\qquad (\varepsilon - H_c)\,\Phi(0) = 0$

and $\qquad W\,\Phi(0) = 0$

assuming the compound states contain no photons.

By iteration we get:

$$\Phi = \Phi(0) + \frac{1}{\varepsilon - H_c - W} I\,\Phi(0) + \frac{1}{\varepsilon - H_c - W} I \frac{1}{\varepsilon - H_c - W} I\,\Phi(0) + \dots$$

$$= \Phi(0) + \frac{1}{\varepsilon' - H_c} I\,\Phi(0) + \dots = \left(1 + \sum_{i=1} \Omega_i\right)\Phi(0)$$

$$\Phi(\varepsilon, 0) = \Phi_\varepsilon$$

$$\mathcal{V} = V_{00} + I_{00} + \sum_\varepsilon \frac{V_0 \Phi_\varepsilon\rangle\langle\Phi_\varepsilon V_0^+}{E^\pm - \varepsilon} +$$

where: $\varepsilon' = \varepsilon - \omega$

So, allowing at most one photon processes:

$$\Phi = \left[1 + \frac{1}{\varepsilon' - H_c} I\right] \Phi(0) = (1 + \Omega_1)\, \Phi(0)$$

Let's study now the effective potential:

$$\mathcal{V} = V_{00} + I_{00} + V_0 \frac{1}{E^\pm - H} V_0^+ + I_0 \frac{1}{E^\pm - H} V_0^+ + V_0 \frac{1}{E^\pm - H} I_0^+ + I_0 \frac{1}{E^\pm - H} I_0^+$$

Let's consider now a typical term:

$$I_0 \frac{1}{E^\pm - H} V_0^+ = \mathop{S}_\varepsilon \frac{I_0 \Phi(\varepsilon)\rangle\langle\Phi(\varepsilon) V_0^+}{E^\pm - \varepsilon} = \mathop{S}_\varepsilon \frac{I_0 \Phi(\varepsilon,0)\rangle\langle\Phi(\varepsilon,0) V_0^+}{E^\pm - \varepsilon} +$$

$$+ \mathop{S}_\varepsilon \frac{I_0 \Omega_1 \Phi(\varepsilon,0)\rangle\langle\Phi(\varepsilon,0) V_0^+}{E^\pm - \varepsilon} + \mathop{S}_\varepsilon \frac{I_0 \Phi(\varepsilon,0)\rangle\langle\Phi(\varepsilon,0) \Omega_1^+ V_0^+}{E^\pm - \varepsilon} + \dots$$

So: calling $\Phi(\varepsilon, 0) \equiv \Phi(\varepsilon)$

$$\mathcal{V} = \underset{0}{V_{00}} + \underset{1}{I_{00}} + \mathop{S}_\varepsilon \underset{0}{\frac{V_0 \Phi(\varepsilon)\rangle\langle\Phi(\varepsilon) V_0^+}{E^\pm - \varepsilon}} + \mathop{S}_\varepsilon \underset{1}{\frac{I_0 \Phi(\varepsilon)\rangle\langle\Phi(\varepsilon) V_0^+}{E^\pm - \varepsilon}} + \mathop{S}_\varepsilon \underset{1}{\frac{V_0 \Phi(\varepsilon)\rangle\langle\Phi(\varepsilon) I_0^+}{E^\pm - \varepsilon}}$$

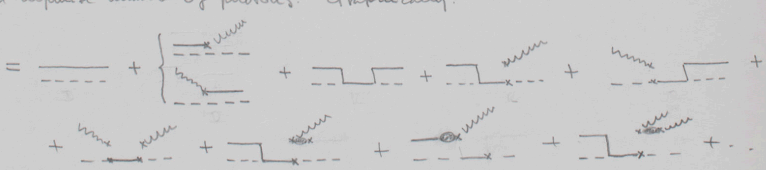
order in I

$$+ \mathop{S} \underset{2}{\frac{I_0 \Phi\rangle\langle\Phi I_0^+}{E^\pm - \varepsilon}} + \mathop{S} \underset{1}{\frac{V_0 \Omega_1 \Phi\rangle\langle\Phi V_0^+}{E^\pm - \varepsilon}} + \mathop{S} \underset{1}{\frac{V_0 \Phi\rangle\langle\Phi \Omega_1^+ V_0^+}{E^\pm - \varepsilon}} + \mathop{S} \underset{2}{\frac{I_0 \Omega_1 \Phi\rangle\langle\Phi V_0^+}{E^\pm - \varepsilon}}$$

$$+ \dots\dots$$

We have written the operator $\mathcal{V}$ as an infinite sum of terms each conecting states with a definite number of photons: Graphically:

$\mathcal{V} =$



$$\frac{\Phi_\varepsilon\rangle\langle\Phi_\varepsilon I_0^+}{E^+-\varepsilon} + \frac{\Phi_\varepsilon\rangle\langle\Phi_\varepsilon V_0^+}{E^+-\varepsilon} \rightarrow \frac{V_0\Phi_\varepsilon\rangle\langle\Phi_\varepsilon I_0^+}{E^+-\varepsilon} + \frac{V_0\Phi_\varepsilon\rangle\langle\Phi_\varepsilon V_0^+}{E^+-\varepsilon} + \frac{I_0\Phi_\varepsilon\rangle\langle\Phi_\varepsilon I_0^+}{E^+-\varepsilon} + \frac{I_0\Phi_\varepsilon\rangle\langle\Phi_\varepsilon V_0^+}{E^+-\varepsilon}$$

$$\rightarrow \frac{V_0\Phi(0)+V_0\Phi(1)\rangle\langle\Phi(0)I_0^+ + \Phi(1)I_0^+}{E^+-\varepsilon} + \cdots$$

$$= \frac{V_0\Phi(0)\rangle\langle\Phi(0)I_0^+}{E^+-\varepsilon} + \frac{V_0\Phi(1)\rangle\langle\Phi(0)I_0^+}{E^+-\varepsilon} + \frac{V_0\Phi(0)\rangle\langle\Phi(1)I_0^+}{E^+-\varepsilon} + \frac{V_0\Phi(1)\rangle\langle\Phi(1)I_0^+}{E^+-\varepsilon} + \cdots$$

$$W_{00} \quad \cancel{W_{11}} \quad W_{10} \quad W_{10} \quad (VI)_{00} \quad \cancel{(VI)_{11}} \quad \cancel{(VI)_{01}} \quad \cancel{(VI)_{10}} \quad (IV)_{00} \quad \cancel{(IV)_{11}} \quad \cancel{(IV)_{01}} \quad \cancel{(IV)_{10}}$$

$$a_i b_j \rightarrow (a_{1i}+a_{2i})(b_{1j}+b_{2j}) =$$

$$\underline{ab} = (\underline{a_1+a_2})(\underline{b_1+b_2}) = \underline{a_1b_1} + \underline{a_2b_1} + \underline{a_1b_2} + \underline{a_2b_2}$$

Pure elastic scattering. Isolated resonance.

$$E \simeq \varepsilon_r$$

$$\mathcal{V} = \mathcal{V}_r + \frac{(V_0+I_0)\Phi(\varepsilon_r)\rangle\langle\Phi(\varepsilon_r)(I_0^+ + V_0^+)}{E-\varepsilon_r}$$

Let's call:

$$H_0 = T + W + \mathcal{V}_r$$

∴ the wave eq. becomes:

$$(E-H_0)\,u_0 = -\Lambda_r (V_0+I_0)\Phi \qquad (2)$$

where:

$$\Lambda_r = \frac{\langle\Phi(\varepsilon_r)(I_0^+ + V_0^+)u_0\rangle}{E-\varepsilon_r}$$

Solving (2):

$$u_0 = \chi^+ + \Lambda_r \frac{1}{E-H_0}(V_0+I_0)\Phi$$

where:

$$(H_0-E)\chi = 0$$

So we get:

$$\Lambda_r = \frac{\langle\Phi(\varepsilon_r)(I_0^+ + V_0^+)\chi^+\rangle}{E-\varepsilon_r - \langle\Phi(\varepsilon_r)(V_0^+ + I_0^+)\frac{1}{E^+-H_0}(I_0+V_0)\Phi(\varepsilon_r)\rangle}$$

Transition matrix:

$$T_{fi} = T_{fi}^D + T_{fi}^R$$

$$T_{fi}^D = \langle \Upsilon_f \mathcal{V}_r \chi_i^+ \rangle$$

$$T_{fi}^R = \Lambda_r \langle \chi_f^- (V_0+I_0)\Phi(\varepsilon_r)\rangle$$

$$= \frac{\langle\chi_f^-(V_0+I_0)\Phi(\varepsilon_r)\rangle\langle\Phi(\varepsilon_r)(I_0^+ + V_0^+)\chi_i^+\rangle}{E-E_r + i\pi\sum_f |\langle\chi_f^-(V_0+I_0)\Phi(\varepsilon_r)\rangle|^2}$$

$$\chi^+ = \left[1 + \sum_n \left(\frac{1}{E^+ - T - U_r - W} \mathcal{V}_r(I)\right)^n\right] \chi^+(n)$$

where:

$$(T + W - E)\Upsilon = 0$$

$$E_r = \varepsilon_r + \langle \Phi (V_0^+ + I_0^+) \mathcal{P} \frac{1}{E - H} (I_0 + V_0) \Phi \rangle$$

Let's study now the χ's:

$$(H_0 - E)\chi = (T + W + \mathcal{V}_r - E)\chi = 0$$

$$\mathcal{V}_r = U_r + \mathcal{V}_r(I) = V_{00} + \mathop{S}_{\substack{\varepsilon \\ \text{except} \\ \varepsilon = \varepsilon_r}} \frac{V_0 \Phi(\varepsilon) \rangle\langle \Phi(\varepsilon) V_0^+}{E^\pm - \varepsilon} + \mathcal{V}_r(I)$$

$$\mathcal{V}_I(I) = I_{00} + \mathcal{V}_r'(I)$$

$$\mathcal{V}_r'(I) = \sum_{\varepsilon \neq \varepsilon_r} \frac{V_0 \Phi_\varepsilon \rangle\langle \Phi_\varepsilon I_0^+}{E^\pm - \varepsilon_r} + \sum_{\varepsilon \neq \varepsilon_r} \frac{I_0 \Phi_\varepsilon \rangle\langle \Phi_\varepsilon V_0^+}{E^\pm - \varepsilon_r} + \dots,$$

$$\therefore (T + W + U_r - E)\chi = -\mathcal{V}_r(I)\chi$$

$$\chi^+ = \chi^+(n) + \frac{1}{E^\pm - T - U_r - W} \mathcal{V}_r(I) \chi^+$$

where: $(T + W + U_r - E)\chi(n) = 0$

and $\chi(n)$ contains n photons.

By iteration:

$$\chi^+ = \chi^+(n) + \frac{1}{E^\pm - T - U_r - W} \mathcal{V}_r(I) \chi^+(n) + \dots$$

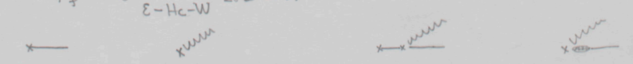
$$= \left[1 + \sum_i T_i^+(I)\right] \chi^+(n)$$

Then:

$$\langle \chi_f^- (V_0 + I_0) \Phi(\varepsilon_r) \rangle = \langle \chi_f^-(n) \left[1 + \sum_i T_i^-(I)\right]^+ (V_0 + I_0)\left(1 + \sum_i \Omega_i\right) \Phi(\varepsilon_r, 0) \rangle =$$

$$= \langle \chi_f^-(0) V_0 \Phi(\varepsilon_r) \rangle + \langle \chi_f^-(1) I_0 \Phi(\varepsilon_r) \rangle + \langle \chi_f^-(1)\, I_{00} \frac{1}{E^\pm - T - U_r - W} V_0 \Phi(\varepsilon_r) \rangle +$$

$$+ \langle \chi_f^-(1) V_0 \frac{1}{\varepsilon - H_c - W} I_0 \Phi(\varepsilon_r) \rangle + \dots$$



So we get, for direct effects:

$$T^D_{fi} = \langle \Upsilon_f \mathcal{V}_r \chi_i^+ \rangle = \langle \Upsilon_f (\mathcal{U}_r + \mathcal{V}_r(I)) \chi_i^+ \rangle = \langle \Upsilon_f \mathcal{U}_r \chi_i^+(n) \rangle + \langle \chi_f^-(n) \mathcal{V}_r(I) \chi_i^+ \rangle$$

$$= \langle \Upsilon_f \mathcal{U}_r \chi_i^+(n') \rangle + \langle \chi_f^-(n') [I_{00} + \mathcal{V}_r'(I)] [1 + \sum_n (\frac{1}{E^+ - T - \mathcal{U}_r - W} \mathcal{V}_r(I))^n] \chi_i^+(n) \rangle$$

At first order in the electromagnetic potential:

$$T^D_{fi} = \langle \Upsilon_f \mathcal{U}_r \chi_i^+(0) \rangle + \langle \chi_f^-(n') [I_{00} + \mathcal{I}] \chi_i^+(n) \rangle$$

where:

$$\mathcal{I} = \sum_{\varepsilon \neq \varepsilon_r} \frac{(I_0 + V_0 \cancel{\frac{1}{\varepsilon' - H_c}} I) \Phi_\varepsilon \rangle \langle \Phi_\varepsilon V_0^+}{\varepsilon_r - \varepsilon} + \sum_{\varepsilon \neq \varepsilon_r} \frac{V_0 \Phi_\varepsilon \rangle \langle \Phi_\varepsilon (I_0^+ + I \cancel{\frac{1}{\varepsilon' - H_c}} V_0^+)}{\varepsilon_r - \varepsilon}$$

This expression describes direct reactions, photoemission, radiative capture etc.

For radiative capture we have:

$$T^{D\,capt}_{fi} = \langle \chi_f(1) [I_{00} + \mathcal{I}'] \chi_i^+(0) \rangle$$

$$\mathcal{I}' = \sum_{\varepsilon \neq \varepsilon_r} \frac{I_0 \Phi_\varepsilon \rangle \langle \Phi_\varepsilon V_0^+}{\varepsilon_r - \varepsilon}$$

Graphically:

For resonant effects we get:

$$T^R_{fi} = \frac{\langle}{}$$

Universidad Nacional Autónoma de México

Direct Radiative Proton Capture Reactions in Cerium and Bismuth

P. J. Daly, J. R. Rook and P. E. Hodgson.

Nucl. Phys. $\underline{56}$, 331 (1964).

Mathematical Formulation.

The present calculation used essentially the same formalism as Lane and Lynn. They used the following expression for the direct capture cross-section:

$$\sigma = \frac{8\pi \bar{e}^2 M'}{3k'\hbar^2} \sum_{\ell\ell'} \left(\frac{E_\gamma}{\hbar c} \right)^2 (\ell + \ell' + 1) | I_{\ell\ell'} |^2 ,$$

where ℓ' and ℓ are the angular momenta before and after the transition, $\bar{e}$ the effective nucleon charge, M' the reduced nucleon mass, k' the incident wave number, E_γ the photon energy, R the nuclear radius and $I_{\ell\ell'}$ the dipole integral

$$I_{\ell'\ell} = \int_0^\infty \psi_{\ell'}(r) \psi_{N\ell}(r) r^3 dr$$

where $\psi_{\ell'}(r)$ is the radial wave function of angular momentum ℓ' of the incident particle and $\psi_{N\ell}(r)$ is the wave function of the nucleon in its final bound state. The E1 selection rule permits only those transitions with $\ell' - \ell = \pm 1$.

A computer programme was written to evaluate the above expression for all possible final states of the captured nucleon.

The wave functions $f_\ell(r)$ and $f_{n\ell}(r)$ were obtained by integrating the radial wave equation with the appropriate boundary conditions. The nuclear potential experienced by the incident particle has the form

$$V(r) = V_c(r) - (U + iW)\, f(r)$$

where $V_c(r)$ is the Coulomb potential, taken to be due to a uniformly charged sphere, U and W are the real and imaginary potential depths, $f(r) = \left[1 + \exp\left(r - r_0 A^{\frac{1}{3}}\right)/a\right]^{-1}$ is the Saxon-Woods form factor, with $r_0 = 1.25$ fm. and the diffuseness parameter $a = 0.65$ fm. Spin-orbit forces were not taken into account in the calculation of the wave function of the incident particle.

The potential used to calculate the energies of the single-particle states wave-functions has the form

$$V(r) = V_c(r) + U f(r) + U_s \frac{1}{r}\frac{df}{dr}\, \underline{\ell}\cdot\underline{s}$$

where U_s is the spin-orbit potential, $\underline{\ell}\cdot\underline{s}$ the spin-orbit operator and the other terms are as before. The potential depth U was adjusted for each state to give the experimentally known binding energy.

LA DESCRIPCION DEL COMPORTAMIENTO DE LOS SISTEMAS MICROSCOPICOS

0 - Sistemas macroscópicos y microscópicos.

1- Descripción clásica del movimiento: la Mecánica clásica.

2- La descripción contemporánea: la Mecánica cuántica.

3- Objetivos de la M.C. Métodos de lograrlo.

4- Idea fundamental: el campo ψ

5- Determinación de datos observables: la interpretación estadística.

6- Descripción de partículas: segunda cuantización.

7- Condiciones impuestas por la T. de la relatividad.

8- Consideraciones sobre el mundo submicroscópico: las partículas elementales.

sica congrega
de América

a- Hipótesis sobre la Teoría cuántica relativista.
Ecs. de cds. Su transf. la cond. espectral y
la existencia del vacío.

1- Hipótesis sobre el dominio y continuidad del campo.

2- Leyes de transf. del campo.

3- Conmutatividad local (Causalidad microscópica)

4- Completez asintótica.

LA TEORIA DE LOS PROCESOS RADIATIVOS EN LAS REACCIONES NUCLEARES.

11-III-1966.

I. INTRODUCCION

El descubrimiento de los procesos radiativos en las reacciones nucleares es tan viejo como la Física Nuclear. Chadwick y Goldhaber[1], irradiando una cámara llena de deuterio con γ's obtenidos del ThC'', descubrieron la fotodesintegración del deuterón y de sus mediciones determinaron la masa del neutrón. Pero la teoría de estos procesos es muy reciente.

Los fenómenos más importantes en este campo son la captura radiativa de nucleones y el efecto fotonuclear. Las primeras explicaciones del primer efecto fueron dadas dentro del modelo del núcleo compuesto[2]. Cuando un nucleón entra en un núcleo para formar un núcleo compuesto, existe la probabilidad $\Gamma_{rad}/\hbar$ por unidad de tiempo, de que el estado compuesto se desintegre mediante la emisión de un fotón. La anchura radiativa se puede expresar en términos de multipolos como

$$\Gamma_{rad} = \sum_{l=1}^{\infty} (\Gamma_{El} + \Gamma_{Ml})$$

en donde Γ_{El} y Γ_{Ml} son las anchuras radiativas correspondientes al multipolo eléctrico o magnético, respectivamente, de orden l. La anchura total, que aparece en las fórmulas de Breit-Wigner es entonces:

$$\Gamma_{tot} = \sum_{\alpha} \Gamma_{\alpha} + \Gamma_{rad}$$

en donde Γ_{α} es la anchura correspondiente al canal α.

La descripción del efecto fotonuclear fué primeramente formulada para la fotodesintegración del deuterón, la cual se explica como una transición del estado base del deuterón a un estado final con un nucleón en el continuo. Aunque los demás casos de fotodesintegración pueden interpretarse de la misma manera, la teoría del efecto fotonuclear se construyó también sobre el modelo del núcleo compuesto[3]. El proceso se divide en dos partes: primeramente se considera la absorción de un fotón y después se estudia la desintegración del núcleo compuesto. Con esta consideración, la sección para la fotoemisión de la partícula b puede escribirse como:

$$\sigma(\gamma, b) = \sigma_c(\gamma)\, G_b ,$$

en donde $\sigma_c(\gamma)$ es la sección de absorción de radiación electromagnética y G_b es la probabilidad de desintegración del estado compuesto por emisión de la partícula b.

Desde la época en la que estos modelos fueron propuestos, se encontró que las predicciones de ellos no eran muy satisfactorias en comparación con los resultados experimentales. En el efecto fotonuclear se encontró una fuerte asimetría en la distribución angular de las partículas emitidas[4], lo cual condujo a la sugestión de mecanismos directos de fotoproducción[5]. El descubrimiento de las resonancias gigantes en el efecto fotonuclear[6] resultó inexplicable en términos de estados compuestos y su explicación ha requerido modelos muy sofisticados[7]. En la descripción de la captura radiativa de nucleones también se encontraron dificultades y hubo necesidad de introducir también mecanismos directos[8].

Otro fenómeno radiativo conocido es el bremsstrahlung nuclear[9], el cual a parte de su interés intrínseco, ha llamado la atención por aportar datos para el entendimiento de las reacciones nucleares[10]. La teoría de este fenómeno se ha construido independientemente de la teoría de las reacciones nucleares, siguiendo el modelo del bremsstrahlung ordinario[11].

II. TEORIAS SOBRE PROCESOS RADIATIVOS EN REACCIONES NUCLEARES

Las teorías propuestas para la descripción de los procesos radiativos en las reacciones nucleares han sido formuladas como extensiones de las teorías de las reacciones nucleares. Se revisarán brevemente las existentes denominándolas según la teoría de las reacciones nucleares sobre la cual se han construido.

1- La teoría de Wigner de la matriz R.

Tanto la teoría de las reacciones nucleares basada en la matriz R[11 bis], como su extensión a procesos radiativos se encuentra resumida en el artículo de Lane & Thomas[12]. En este hay una sección, titulada "La inclusión de canales con fotones" en la que se trata este tema. Su propósito es extender las fórmulas para procesos en resonancia al caso en que intervengan fotones. Se calculan las anchuras para fotones y se prueba que intervienen en la anchura total del estado compuesto. La extensión para procesos directos ha sido hecha por Lane & Lynn[13], desarrollando los estados de núcleo compuesto $X_{\lambda(JM)}$ en términos de un conjunto completo formado por productos del tipo $u(r_c)\,\chi_{c(JM)}$, en donde $u(r)$ es ~~el~~ un estado de una partícula y χ_c el estado del núcleo residual.

2- El formalismo de Kapur - Peierls.

La versión moderna de ~~esta~~ la teoría de las reacciones nucleares de Kapur-Peierls[13 bis] se encuentra resumida en un artículo de Brown publicado en el Rev. of. Mod. Phys en 1959[14]. A partir de los trabajos sobre el modelo óptico, se insiste en los mecanismos de interacción directa y Brown, en este artículo, formula una extensión a la teoría de las reacciones nucleares en la cual se describe tanto la captura radiativa como el efecto fotonuclear. El método usado consiste en proponer las funciones de onda del sistema como[15]:

$$\Psi(r,\xi) = \sum_{\alpha j} S_{\gamma\alpha j}\, \chi_j(\xi)\, \psi^{+}_{\alpha j}(r) \quad , \qquad \text{(región exterior)}.$$

en donde $S_{\gamma\alpha j}$ es la amplitud para absorción de un fotón con emisión de un nucleón en el canal α dejando el nucleo residual en el estado j y

$$\Psi(r,\xi) = \Psi_0(r,\xi) + \sum_{p} a_p \Phi^{(p)}(r,\xi) \qquad \text{(región interior)}$$

en donde $\Phi^{(p)}$ son las funciones de estado compuesto y Ψ_0 representa el nucleo inicial y un fotón presente. Aunque Brown exhibe efectos directos, el énfasis de su trabajo está también en el cálculo de las anchuras para fotones. En un artículo posterior, Brown[16] introduce lo que llama procesos semidirectos, encontrando para la amplitud de fotoemisión de una partícula, la expresión:

$$S_{\alpha m} = \langle \psi_\alpha(r)\, \chi_0(\xi) \,|\, I \,|\, \psi_m(r)\, \chi_0(\xi) \rangle$$

en donde

$$I = \mathcal{E} + (V-\bar{V})\, \frac{1}{E_0 - H + \hbar\omega}\, \mathcal{E} + \mathcal{E}\, \frac{1}{E_0 - H - \hbar\omega}\, (V-\bar{V})$$

y χ_0 es el estado base del nucleo residual, ψ_m y ψ_α las funciones de onda de la partícula en el estado m y en el contínuo, respectivamente, H el hamiltoniano total en el que $V = V(r,\xi)$ es la interacción entre la partícula y el nucleo residual, $\bar{V}$ un potencial común efectivo, $\hbar\omega$ la energía del fotón y

$$\mathcal{E} = \sum_{i=1}^{A} \mathcal{E}(\xi_i) + \mathcal{E}(r)$$

el operador de dipolo.

3- La teoría unificada de las reacciones nucleares.

Uno de los resultados mas sobresalientes de la teoría unificada de las reacciones nucleares, propuesta por Feshbach[17] es la de obtener, de manera natural, la descripción de procesos directos y resonantes. En términos

de la amplitud de transición T para la reacción $\alpha \to \beta$, este hecho se expresa como:

$$T(\beta|\alpha) = T_P(\beta|\alpha) + T_R(\beta|\alpha)$$

en donde T_R es una formula del tipo Breit-Wigner. La extensión para procesos radiativos[18] se obtiene interpretando el Hamiltoniano total H como:

$$H = H^{N} + H^{\gamma} + H^{\gamma N}$$

en donde H^{N} es el hamiltoniano de los nucleones, H^{γ} el del campo de radiación y $H^{\gamma N}$ describe la interacción entre nucleones y radiación que en la aproximación de un fotón es:

$$H^{\gamma N} \sim e \sum_i \underline{j}_i \cdot \underline{A}_i$$

en donde $\underline{j}_i$ es la corriente debida al proton i y $\underline{A}_i$ el potencial vectorial.

El proyector sobre los canales abiertos es

$$\underline{P} = \underline{P}_0 + \underline{P}_1$$

si se restringe a estados en los que no haya mas de un fotón presente. En el artículo en el que se presenta esta extensión[18], se estudia especialmente el término resonante. Se prueba que la interacción electromagnética no modifica las ~~canales de partículas~~ anchuras para emisión y absorción de partículas y para las anchuras radiativas se encuentra que el elemento de matriz correspondiente resulta:

$$\langle \psi_\beta^{(-)}\, H_{PQ}\, \Phi_s \rangle = \langle \psi_\beta^{(-)}(1) \,|\, I^{\gamma N} \,|\, \Phi_{s0}(0) \rangle$$

en donde: $I^{\gamma N} = H^{\gamma N} + \underline{P}_1 H' \underline{P}_0 \dfrac{1}{E^{+} - H'_N}\, H^{N}_{PQ}$

~~en donde~~ y ψ_β^{-} es la función de onda en el canal β, Φ_{s0} es la función de onda del estado compuesto y H' la parte del hamiltoniano total que opera sobre las funciones de onda de los canales abiertos. El segundo termino incluye procesos semejantes a los semidirectos de Brown y fué estudiado por Shakin[19] para el efecto fotonuclear.

Términos de este tipo existen también en los procesos directos y se encuentran implícitos en la expresión (2.12) de la Ref. 18. Es fácil exhibirlos explícitamente.[20]

III. FORMALISMO GENERAL PARA LA DESCRIPCION DE PROCESOS RADIATIVOS EN REACCIONES NUCLEARES.

Se presentará ahora un tratamiento sistemático de los procesos radiativos en las reacciones nucleares. Para dar una presentación clara se dará una formulación simplificada. Consideremos N nucleones, interaccionando mediante fuerzas entre pares, en un campo de radiación. Se puede expresar el Hamiltoniano total del sistema en la forma:

$$H = H_{nucl} + \mathcal{H} + I$$

en donde
$$H_{nucl} = \sum_{i=1}^{N} T_i + \sum_{i<j}^{N} V_{ij}$$

$$\mathcal{H} = \sum_{\underline{k}} \hbar\omega\, b_{\underline{k}}^{+} b_{\underline{k}}$$

$$I = -\frac{e}{mc} \sum_{prot} \underline{p_i} \cdot \underline{A} \qquad \text{y} \qquad \underline{A} = \sum_{\underline{k}} (b_{\underline{k}} A_{\underline{k}} + b_{\underline{k}}^{+} A_{\underline{k}}^{*})$$

siendo b y b^{+} los operadores de aniquilación y creación de fotones respectivamente.

La función de onda del sistema completo:

$$\Psi = \Psi(1, 2, \ldots, N, \underline{b}, b^{+})$$

satisface la ecuación:
$$H\Psi = E\Psi$$

Nuestro modelo simplificado consiste en expresar el Hamiltoniano como:

$$H = H_{N-1}(\xi) + T(r) + \mathcal{H} + V(r,\xi) + I(r,\xi,b,b^{+}).$$

y usar las eigenfunciones ψ_i:

$$H_{N-1}\psi_i = \epsilon_i \psi_i$$

para escribir la función de ondas como:

$$\Psi = \sum_i U_i(r, b, b^{+})\, \psi_i(\xi).$$

De esta manera podemos obtener, para el canal incidente, la ecuación:

$$(T + \mathcal{H} + \mathcal{V} - E)\, U_0 = 0$$

en donde el potencial efectivo es:

$$\mathcal{V} = V_{00} + I_{00} + (\underset{\sim}{V_0} + \underset{\sim}{I_0}) \frac{1}{E \pm \underset{\sim}{H}} (\underset{\sim}{I_0}^{+} + \underset{\sim}{V_0}^{+})$$

con:
$$V_{ij} = \langle \psi_i, V\psi_j \rangle \qquad I_{ij} = \langle \psi_i, I\psi_j \rangle$$

$$\underset{\sim}{V_0} = (V_{01}, V_{02}, \ldots) \qquad \underset{\sim}{I_0} = (I_{01}, I_{02}, \ldots)$$

$$\underset{\sim}{H} = \|H_{ij}\| \qquad H_{ij} = (T + \mathcal{H} + \epsilon_i)\delta_{ij} + V_{ij} + I_{ij} \quad \text{para } i,j \neq 0.$$

Introduciendo eigenfunciones de $\underset{\sim}{H}$:

$$\underset{\sim}{H}\,\underset{\sim}{\Phi}_\varepsilon = \varepsilon\,\underset{\sim}{\Phi}_\varepsilon, \quad \text{o sea:} \quad (\underset{\sim}{H_c} + \mathcal{H} + I - \varepsilon)\,\underset{\sim}{\Phi}_\varepsilon = 0$$

con
$$\underset{\sim}{H_c} = \|h_{ij}\| \qquad h_{ij} = (T + \epsilon_i)\delta_{ij} + V_{ij}$$

que es el operador $\underset{\sim}{H}$ de Feshbach; Se obtiene inmediatamente:

$$\Phi_\varepsilon = \Phi(0) + \frac{1}{\varepsilon - \underset{\sim}{H_c} - \mathcal{H}}\, I\, \Phi_\varepsilon$$

en donde
$$(\varepsilon - \underset{\sim}{H_c})\,\Phi(0) = 0 \qquad \text{y} \qquad \mathcal{H}\,\Phi(0) = 0$$

ya que se supone que no hay fotones reales en los estados compuestos.

A 1er orden en I se obtiene:

$$\Phi_\varepsilon = \left(1 + \frac{1}{\varepsilon - \underset{\sim}{H_c} - \mathcal{H}} I\right) \Phi(0) \equiv \Phi_\varepsilon(0) + \Phi_\varepsilon(1)$$

Con ayuda de este resultado se puede probar que para procesos en los que inicialmente, o finalmente, haya un sólo fotón, se obtiene:

$$(\underset{\sim}{V_0} + \underset{\sim}{I_0}) \frac{1}{E^\pm - \underset{\sim}{H}} (\underset{\sim}{I_0^+} + \underset{\sim}{V_0^+}) = \underset{\sim}{V_0} \frac{1}{E^\pm \underset{\sim}{H_c}} \underset{\sim}{V_0^+} + \underset{\sim}{V_0} \frac{1}{a} \underset{\sim}{I_0^+} + \underset{\sim}{I_0} \frac{1}{a} \underset{\sim}{V_0^+} + \underset{\sim}{V_0} \frac{1}{a} I \frac{1}{a} \underset{\sim}{V_0^+}$$

$$+ \cancel{V_0 \tfrac{1}{a} V \tfrac{1}{a} V_0^+} + \underset{\sim}{V_0} \frac{1}{a} I \frac{1}{a} \underset{\sim}{I_0^+} + \underset{\sim}{I_0} \frac{1}{a} I \frac{1}{a} \underset{\sim}{V_0^+} + \underset{\sim}{V_0} \frac{1}{a} I \frac{1}{a} I \frac{1}{a} \underset{\sim}{V_0^+}$$

$$+ \underset{\sim}{V_0} \frac{1}{a} I \frac{1}{a} I \frac{1}{a} \underset{\sim}{I_0^+} + \underset{\sim}{I_0} \frac{1}{a} I \frac{1}{a} I \frac{1}{a} \underset{\sim}{V_0^+} \equiv (VV)_{00} + (VI)_{00} + (IV)_{00}$$

$$+ [(VV)_{01} + (VV)_{10}] + [(VI)_{01} + (VI)_{10}] + [(IV)_{01} + (IV)_{10}] + (VV)_{11}$$

$$+ (VI)_{11} + (IV)_{11}$$

Escribiendo: $\mathcal{V} = \mathcal{V}_0 + \mathcal{V}_I$

en donde
$$\mathcal{V}_0 \equiv V_{00} + (VV)_{00}$$
$$\mathcal{V}_I = I_{00} + (VI)_{00} + (IV)_{00} + \ldots.$$

La matriz de transiciones es:

$$T(f|i) = \langle \phi, u | \mathcal{V} | \mathcal{V}_0^+ \rangle$$

en donde $(T + \mathcal{H} - E)|\phi, u\rangle = 0$

Introduciendo las funciones de onda:

$$(T + \mathcal{H} + \mathcal{V}_0 - E) W_0 = 0$$

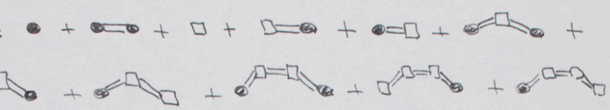
podemos escribir:

$$T(f|i) = \langle \phi, u | \mathcal{V}_0 | W_0^+ \rangle + \langle W_0^- | \mathcal{V}_I | \mathcal{V}_0^+ \rangle$$

El 1er término es esencialmente la matriz de transiciones para reacciones nucleares, obtenida por Feshbach. El segundo término contiene <u>todos</u> los procesos radiativos que contienen un fotón a lo más en sus estados

inicial y final.

Gráficamente se puede escribir:

$\mathcal{V} = \mathcal{V}_0 + \mathcal{V}_I =$ 

y los procesos radiativos se pueden obtener sistemáticamente mediante las siguientes reglas:

línea ondulada = 1 fotón.
línea recta sólida —— = 1 nucleón libre.
línea recta punteada - - - = 1 nucleón ligado.

Reglas: sólo puede tener una raya recta antes y después.
una raya con o sin onda, antes o después, pero una sola onda en total.

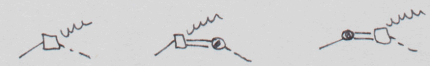
En los dos primeros términos sólo hay rayas sólidas. En los siguientes tiene que haber 1 raya sólida a fuerzas.

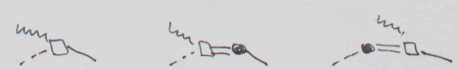
Ejemplos:

Dispersión por un potencial 

Dispersión resonante 

Bremsstrahlung. 

Captura radiativa. 

Efecto fotonuclear. 

Referencias

1 – Chadwick & Goldhaber. Nature 134, 237 (1934)

2 – Blatt & Weisskopf. "Theoretical Nuclear Physics". Wiley (1952) p. 647

3 – Ref. ant. p. 651.

4 – Hirzel & Wäffler. Helv. Phys. Acta 20, 373 (1947).

5 – Courant. Phys. Rev. 82, 703 (1951)
Wilkinson. Physica 22, 1039 (1956)
Brown & Levinger. Proc. Phys. Soc. 71, 733 (1958).

6 – Montalbetti, Goldemberg & Katz. Phys. Rev. 91, 659. (1953)
Fuller & Hayward "Nuclear Reactions" Vol. II, Ed. by Endt & Smith. North Holland Pub. 1962.

7 – Goldhaber & Teller. Phys. Rev. 74, 1046 (1948)
Wilkinson. Ref. 5.
Brown & Bolsterli. Phys. Rev. Lett. 3, 472 (1959).

8 – Cohen Phys. Rev. 100, 206 (1955)
Beck Physica 22, 1164 (1956).

9 – Wilson Phys. Rev. 85, 563 (1952).

10 – Eisberg, Yennie & Wilkinson. Nucl. Phys. 18, 338 (1960).

11 – Kursunoğlu Phys. Rev. 105, 1846 (1957).
Feshbach & Yennie Nucl. Phys. 37, 150 (1962)

11bis – Wigner & Eisenbud Phys. Rev. 72, 29, (1947)

12 – Lane & Thomas. Rev. Mod. Phys. 30, 257 (1958).

13 – Lane. Nucl. Phys. 11, 625 (1959)
Lane & Lynn Nucl. Phys. 11, 646 (1959); 17, 563 (1960).
~~Lane & Lynn Nucl. Phys. 17, 563 (1960).~~

13bis – Kapur & Peierls Proc. Roy. Soc A166, 277 (1937)

14 – Brown Rev. Mod. Phys. 31, 893 (1959).

15 – Brown & Levinger Proc. Phys. Soc. A71, 733 (1958).

16 – Brown Nucl. Phys. 57, 339 (1964).

17 – Feshbach Ann. Phys (N.Y.) 5, 357 (1958); 19, 287 (1962)

18 – Estrada & Feshbach Ann. Phys (N.Y.) 23, 123 (1963)

19 – Shakin Ann. Phys (N.Y.) 22, 54, (1963)

20 – Andrade y Estrada. Rev. Mex. Fís, 14, 183, (1965).

Theory of Radiative Capture in the Resonance Region

A. M. Lane and J. E. Lynn.
Nuclear Physics 17, (1960) 563.

Dispersion Formalism for Radiative Capture.

Particles α' of spin $i_{\alpha'}$ are incident upon nuclei β' of spin $i_{\beta'}$ with relative orbital angular momentum ℓ'. Channel spin S:

$$\underline{s}' = \underline{i}_{\alpha'} + \underline{i}_{\beta'}$$

Total spin J:

$$\underline{J} = \underline{s}' + \underline{\ell}'$$

Wave function $\Psi_{E(JM)}$ of total energy E, spin J and component M in the "channel" $c' \equiv (\alpha', \beta', s', \ell')$:

$$\Psi_{E(JM)} = v'^{\frac{1}{2}} \left[I_{c'}(k'r) - U^{(J)}_{c',c'}\, O_{c'}(k'r) \right] \varphi_{c'(JM)} \qquad (1)$$

where $\varphi_{c'(JM)}$ is the channel function (wave functions of α' and β' coupled to channel spin s' and then coupled to the angular functions $\frac{i^{\ell'}}{r} Y^{(\ell')}_{m}$ to give total spin J.) v' is the velocity in the channel and:

$$\varepsilon' = \frac{1}{2} M' v'^2 = \frac{\hbar^2 k'^2}{2M'} . \qquad (2)$$

where M' is the reduced mass; $I_{c'}$ and $O_{c'}$ in (1) are the spherical Bessel function that

$$I_{c'}(k'r) \rightarrow e^{-i(k'r - \frac{1}{2}\ell'\pi)} \qquad O_{c'}(k'r) \longrightarrow e^{+i(k'r - \frac{1}{2}\ell'\pi)} . \qquad (3)$$

Finally, $U^{(J)}_{c',c'}$ in (1) is the diagonal element of the scattering matrix for total spin J. In the resonance region where the mean level width $\langle \Gamma_\lambda \rangle$ is less than the mean level spacing $\langle D \rangle$, this quantity has the form:

$$U^{(J)}_{c',c'} = e^{-2i\phi_{c'}} \left(1 + i \sum_\lambda \frac{\Gamma_{\lambda c'}}{E_\lambda - E - \frac{1}{2} i \Gamma_\lambda} \right) \qquad (4)$$

where the level shift Δ_λ has been dropped as irrelevant to the present discussion. In (4), $\phi_{c'}$ is the hard-sphere scattering phase angle ($= -k'R$ for s-wave neutrons) E_λ and Γ_λ are the energy and total width of the level λ and $\Gamma_{\lambda c'}$ is the partial width of the level λ for the channel c' which is given by squaring:

$$\Gamma^{\frac{1}{2}}_{\lambda c'} \equiv (2P_{c'})^{\frac{1}{2}} \gamma_{\lambda c'} \qquad (5)$$

Here, $P_{c'}$ is the penetration factor and $\gamma_{\lambda c'}$ is the reduced width amplitude:

$$\gamma_{\lambda c'} \equiv \left(\frac{\hbar^2}{2M'R}\right)^{\frac{1}{2}} \int X_{\lambda(JM)} \varphi^*_{c'(JM)} \, dS \qquad (6)$$

where R is the interaction radius, $X_{\lambda(JM)}$ the wave-function of the compound state, dS indicates integration over all coordinates except that of radial distance of separation of α' and β' which is fixed at the value R.

Inside the radial separation R, the wave function $\Psi_{E(JM)}$ may be expanded as $(\langle\Gamma_\lambda\rangle \ll \langle D\rangle)$

$$\Psi_{E(JM)} = -i\hbar^{\frac{1}{2}} e^{-i\phi_{c'}} \sum_\lambda \frac{\Gamma^{\frac{1}{2}}_{\lambda c'} X_{\lambda(JM)}}{E_\lambda - E - \frac{1}{2} i \Gamma_\lambda} \qquad (7)$$

The cross-section for radiative capture of α' and β' to form the final bound state $\Psi_{f(J_f M_f)}$ has the form:

$$\sigma_{\alpha'\beta',\gamma f} = \frac{\pi}{k'^2} \sum_J \frac{2J+1}{(2i_{\alpha'}+1)(2i_{\beta'}+1)} \sum_{s'l'} \left| U^{(J)}_{\gamma f, c'} \right|^2 \qquad (8)$$

where $U^{(J)}_{\gamma f, c'}$ is the scattering matrix element given by

$$U^{(J)}_{\gamma f, c'} = \left(\frac{16\pi}{9\hbar}\right)^{\frac{1}{2}} k_\gamma^{3/2} \frac{\langle \Psi_{f(J_f)} \| \mathcal{H}^{(1)} \| \Psi_{E(J)} \rangle}{(2J+1)^{\frac{1}{2}}} \qquad (9).$$

The reduced matrix element has been introduced as:

$$\left(\frac{2J_f+1}{2J+1}\right)^{\frac{1}{2}} \sum_{M+q=M_f} \langle JM1q | J_f M_f \rangle \int \Psi^*_{f(J_f M_f)} \mathcal{H}^{(1)}_q \Psi_{E(JM)}$$

where the integration is over all coordinates, the summation is for fixed M_f and $\mathcal{H}^{(1)}_q$ is the dipole operator of component q and k_γ the photon wave number corresponding to photon energy ε_γ.

Substituting (7) in (9), the contribution from $r < R$ to $U^{(J)}_{\gamma f, c'}$ is

$$-ie^{i\phi_c} \sum_\lambda \frac{\Gamma^{\frac{1}{2}}_{\lambda c'} \Gamma^{\frac{1}{2}}_{\lambda \gamma f}}{E_\lambda - E - \frac{1}{2} i \Gamma_\lambda} \qquad (10)$$

where the photon width is:

$$\Gamma^{\frac{1}{2}}_{\lambda\gamma f} = \left(\frac{16\pi}{9}\right)^{\frac{1}{2}} k_\gamma^{\frac{3}{2}} \frac{\langle \Psi_{f(J_f)} \| \mathcal{H}^{(1)} \| X_{(\lambda(J))} \rangle}{(2J+1)^{\frac{1}{2}}}, \qquad (11)$$

the integration being confined to $r < R$.

Now we consider the channel contribution to the integral (9). The second term in (4) gives rise to a term in $U_{\gamma f, c'}$ of the form (10) which can be included in (10) by adding to each $\Gamma^{\frac{1}{2}}_{\lambda\gamma f}$ of (11) a term $\delta \Gamma^{\frac{1}{2}}_{\lambda\gamma f}$. This addition is defined as (11) except that the integral in the matrix element is now taken in the channel $(r > R)$ and $X_{\lambda(JM)}$ is extended as an outgoing wave

$$X_{\lambda(JM)} \rightarrow [X_{\lambda(JM)} (r=R)] \frac{R\, O_{c'}(k'r)}{r\, O_{c'}(k'R)}$$

This addition to the photon width amplitude is complex in general, but is essentially real when $P \ll 1$. The remainder of the channel contribution which comes from the first term in (4) can be written as follows: The final state wave-function in the channel $c_f = (\alpha'\beta' s_f l_f)$ is:

$$\Psi_{f(J_f M_f)} = \sqrt{\frac{2}{R}}\, \theta_{f c_f} \varphi_{c_f(J_f M_f)} \left(\frac{O_{c_f}(k_f r)}{O_{c_f}(k_f R)}\right)$$

where $\theta_{f c_f} \equiv \gamma_{f c_f} \left(\frac{\hbar^2}{MR^2}\right)^{-\frac{1}{2}}$ (Cf. eq. (6)) (13)

The outgoing wave $O_{c_f}(k_f r)$ has the form of a decreasing exponential for a bound final state, k_f being the wave-number corresponding to the binding energy ε_f of α' and β' in this state. The final form for the matrix element (9) is a sum of three parts: a resonant part, which contains an interior contribution and a channel contribution, and a non-resonant part corresponding to the channel integral from hard sphere scattering. Dropping the labels $c'\gamma f$ and (J) on U we have the eqs:

$$U = U(\text{res.}) + U(\text{hard sphere})$$

$$U(\text{res.}) = U(\text{int. res}) + U(\text{ch. res.}) = -ie^{-i\phi_{c'}} \sum_{\lambda} \frac{\Gamma^{\frac{1}{2}}_{\lambda c'} \left(\Gamma^{\frac{1}{2}}_{\lambda\gamma f} + (\delta\Gamma_{\lambda\gamma f})^{\frac{1}{2}}\right)}{E_\lambda - E - \frac{1}{2} i \Gamma_\lambda} \qquad (14)$$

$$U(\text{hard sph.}) = \sqrt{\frac{16\pi}{9\hbar v'}}\, k_\gamma^{\frac{3}{2}} \sum_{c_f} \theta_{fc_f} \frac{\langle r\varphi_{c_f(J_f)} \| H^{(1)} \| r\varphi_{c'(J)} \rangle}{(2J+1)^{\frac{1}{2}}}\, 2ik' R^{\frac{5}{2}} e\, I^{(\ell'\ell_f)}_{HS} \qquad (15)$$

where

$$I^{(\ell'\ell_f)}_{HS} = \frac{\sqrt{2}}{R^3 e} \int \frac{O_{c_f}(k_f r)}{O_{c_f}(k_f R)}\, h^{(1)} \psi_{HS}(r)\, r\, dr. \qquad (16)$$

$\psi_{HS}(r)$ is the hard sphere wave-function for incident energy ε'

$$\psi_{HS} = \frac{1}{2k'r}\left(I_{c'} - e^{-2i\phi_{c'}} O_{c'}\right) \qquad (17)$$

The dipole operator $\mathcal{H}^{(1)}_q$ has been split into the angle-intrinsic spin part $H^{(1)}_q$ and the radial part (of dimensions of charge times length) $h^{(1)}$:

$$\mathcal{H}^{(1)}_i = h^{(1)} H^{(1)}_q \qquad (18).$$

For E1 transitions

$$h^{(1)}(r) = \bar{e}\, r \quad , \quad H^{(1)}_q = Y^{(1)}_q(\Omega)$$

where $\bar{e}$ is the effective charge ($-\frac{Ze}{A}$ for neutrons, $\frac{Ne}{A}$ for protons).

Since

$$r\varphi_{c(J)} = \cancel{\sum_{m_\ell=-\ell}^{\ell'} \sum_{m_s=-s}^{s}} = \sum_{\substack{m_{\alpha'}, m_{\beta'} \\ m_s, m}} \langle i_{\alpha'} i_{\beta'} m_{\alpha'} m_{\beta'} | s\, m_s \rangle \langle s\, \ell\, m_s\, m | J M \rangle \,\cdot$$

$$\cdot\, \psi_{\alpha' i_{\alpha'} m_{\alpha'}}(q_1)\, \psi_{\beta' i_{\beta'} m_{\beta'}}(q_2)\, i^{\ell}\, Y^{\ell}_m(\Omega)$$

and

$$\langle \Psi_{f(J_f)} \| \mathcal{H}^{(1)} \| \Psi_{E(J)} \rangle = \sqrt{\frac{2J_f+1}{2J+1}} \sum_{M+q=M_f} \langle J\, 1\, M\, q | J_f M_f \rangle \int \Psi^{*}_{f(J_f)} \mathcal{H}^{(1)}_q \Psi_{E(J)}$$

we get:

$$\frac{\langle r\varphi_{c_f(J_f)} \| H^{(1)} \| r\varphi_{c'(J)} \rangle}{\sqrt{2J+1}} = \frac{\sqrt{2J_f+1}}{2J+1} \sum_{\substack{M q\, m_{\alpha_f} m_{\beta_f} \mu_f m_f \\ m_\alpha m_\beta \mu m}} \langle J\, 1\, M q | J_f M_f \rangle \langle i_{\alpha_f} i_{\beta_f} m_{\alpha_f} m_{\beta_f} | S_f \mu_f \rangle$$

$$\langle S_f \mu_f \ell_f m_f | J_f M_f \rangle \langle i_\alpha i_\beta m_\alpha m_\beta | S \mu \rangle \langle S \ell \mu m | J M \rangle (-)^{\ell_f}\, i^{\ell+\ell_f} \,\cdot$$

$$\delta_{\alpha_f \alpha}\, \delta_{i_{\alpha_f} i_\alpha}\, \delta_{m_{\alpha_f} m_\alpha}\, \delta_{\beta_f \beta}\, \delta_{i_{\beta_f} i_\beta}\, \delta_{m_{\beta_f} m_\beta} \int Y^{*\ell_f}_{m_f}\, Y^1_q\, Y^{\ell}_m =$$

$$= \sqrt{\frac{3}{4\pi}}\, (-)^{\frac{1}{2}(\ell-\ell_f-1)+J_f-J} \sqrt{2\ell+1}\, \sqrt{2J_f+1}\, \delta_{SS_f} \langle \ell\, 1\, 0\, 0 | \ell_f\, 0 \rangle\, W(1\ell_f J S; \ell J_f) =$$

$$= \sqrt{\frac{3}{8\pi}}\, (-)^{\frac{1}{2}(\ell-\ell_f-1)+J_f-J} \sqrt{\frac{\ell+\ell_f+1}{2\ell+1}}\; U(1\ell_f J S; \ell J_f)\, \delta_{S,S_f} \qquad (19).$$

where:

$$\sqrt{\ell+\ell_f+1} = \begin{cases} \sqrt{2(\ell+1)} & \text{for } \ell_f = \ell+1. \\ \sqrt{2\ell} & \text{for } \ell_f = \ell-1. \end{cases}$$

Inserting (15) in (8) and using (19), we find, for either hard sphere or potential capture

$$\sigma_{\alpha'\beta',\gamma_f} = \frac{8}{3}\pi e^2 \frac{R^5}{\hbar v'} k_\gamma^3 \sum_{\ell'\ell_f S} \frac{(2J_f+1)(\ell_f+\ell'+1)}{(2\ell_f+1)(2i_{\alpha'}+1)(2i_{\beta'}+1)}\, \theta_{fc_f} |I^{(\ell',\ell_f)}|^2$$

after a little Racah algebra has been used.

-1-

$$r\varphi_{c'(J)} = \sum_{m_\ell=-\ell}^{\ell} \sum_{m_s=-s}^{s} \langle \ell s m_\ell m_s | \ell s J M\rangle i^\ell Y^\ell_{m_\ell} \chi_{s,m_s}$$

$$\frac{\langle r\varphi_{c_f(J_f)} \| H^{(1)}_q \| r\varphi_{c'(J)}\rangle}{(2J+1)^{\frac{1}{2}}} = \frac{(2J_f+1)^{\frac{1}{2}}}{(2J+1)} \sum_{M+q=M_f} \sum_{m_\ell m_s} \sum_{m'_\ell m'_s} \langle JM1q|JMJ_fM_f\rangle$$

$$\langle \ell s m_\ell m_s | \ell s J M\rangle \langle \ell' s' m'_\ell m'_s | \ell' s' J M\rangle i^{\ell-\ell'} \langle \chi_{s',m'_s} \cdot \chi_{s,m_s}\rangle \int Y^{*\ell'}_{m'_\ell} Y^{(1)}_q Y^\ell_{m_\ell}\, d\tau$$

$$= \frac{(2J_f+1)^{\frac{1}{2}}}{2J+1} \sum_{M+q=M_f} \sum_{m_\ell m'_\ell m_s} i^{\ell-\ell'} \langle JM1q|JMJ_fM_f\rangle \langle \ell s m_\ell m_s | \ell s J M\rangle$$

$$\langle \ell' s m'_\ell m_s | \ell s J M\rangle \delta_{ss'} \cdot \sqrt{\frac{3}{4\pi}} \cdot \left(\frac{2\ell+1}{2\ell'+1}\right)^{\frac{1}{2}} \langle \ell 1 m_\ell q | \ell 1 \ell' m_{\ell'}\rangle \langle \ell 1 0 0 | \ell 1 \ell' 0\rangle$$

$$\sum_{M m_s} \langle \ell s m_\ell m_s | \ell s J M\rangle \langle \ell' s m'_\ell m_s | \ell s J M\rangle = \frac{2J+1}{2\ell+1} \delta_{\ell' m_{\ell'}, \ell m_\ell}$$

$$\frac{\langle \| \| \rangle}{(\ )^{\frac{1}{2}}} = \frac{(2J_f+1)^{\frac{1}{2}}}{(2J+1)} \sum_{m_\ell m_{\ell'}} i^{\ell-\ell'} \langle JM1q|JMJ_fM_f\rangle \delta_{ss'} \sqrt{\frac{3}{4\pi}} \left(\frac{2\ell+1}{2\ell'+1}\right)^{\frac{1}{2}} \frac{2J+1}{2\ell+1} \delta_{\ell' m_{\ell'}, \ell m_\ell}$$

$$\langle \ell 1 m_\ell q | \ell 1 \ell' m_\ell\rangle \langle \ell 1 0 0 | \ell 1 \ell' 0\rangle = \qquad q=1$$

$$= \sqrt{\frac{3}{4\pi}} \frac{(2J_f+1)^{\frac{1}{2}}}{(2\ell'+1)^{1/2}(2\ell+1)^{1/2}} \delta_{ss'} \sum_{m_\ell} \langle JM1q|JMJ_fM_f\rangle \langle \ell 1 m_\ell q | \ell 1 \ell m_\ell\rangle \langle \ell 1 0 0 | \ell 1 \ell 0\rangle$$

Note:

$$U(abcd, ef) = (2e+1)^{\frac{1}{2}}(2f+1)^{\frac{1}{2}} W(abcd, ef)$$

$$C(j_1 j_2 j; m_1 m_2 m) = \langle j_1 j_2 m_1 m_2 | j m\rangle$$

$$\sqrt{\frac{3}{8\pi}} (-)^{\frac{1}{2}(\ell'-\ell_f-1)+J_f-J} \frac{(\ell'+\ell_f+1)^{\frac{1}{2}}}{\cancel{(2\ell'+1)^{\frac{1}{2}}}} \cancel{(2\ell'+1)^{\frac{1}{2}}} (2J_f+1)^{\frac{1}{2}} W(1\ell_f J s', \ell' J_f) \delta_{s's_f}$$

$$= \sqrt{\frac{3}{8\pi}} (-)^{\frac{1}{2}(\ell'-\ell_f-1)+J_f-J} (\ell'+\ell_f+1)^{\frac{1}{2}} (2J_f+1)^{\frac{1}{2}} W(1\ell_f J s', \ell' J_f) \delta_{s's_f}$$

$$C(j_1 j_2 j'; m_1 m_2 m') \qquad m_2 = m' - m_1 \qquad \sum_{m_1 m_2}$$

$$\langle j_1 j_2 m_1 m_2 | JM\rangle = (-)^{j_2+J+m_2+M} \sqrt{\frac{2J+1}{2j_2+1}} \langle J j_1 -M m_1 | j_2 -m_2\rangle$$

$$\begin{pmatrix} j_1 & j_2 & J \\ m_1 & m_2 & -M \end{pmatrix} = \frac{(-)^{j_1-j_2+M}}{\sqrt{2J+1}} \langle j_1 j_2 m_1 m_2 | JM\rangle$$

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ J_1 & J_2 & J_3 \end{Bmatrix} = (-)^{j_1+j_2+J_1+J_2} W(j_1 j_2 J_2 J_1; j_3 J_3)$$

$$\sum_{\substack{M_1 M_2 M_3 \\ m_1 m_2}} (-)^{\cancel{J_1}+\cancel{J_2}+\cancel{J_3}+M_1+M_2+M_3} (-)^{\cancel{J_1}-\cancel{J_2}-m_3} (-)^{\cancel{J_2}-\cancel{J_3}-m_1} (-)^{J_3-\cancel{J_1}-m_2} (-)^{j_1-j_2-m'_3}$$

$$\frac{1}{\sqrt{2j_3+1}} \langle J_1 J_2 M_1 -M_2 | j_3 -m_3\rangle \frac{1}{\sqrt{2j_1+1}} \langle J_2 J_3 M_2 -M_3 | j_1 -m_1\rangle$$

$$\frac{1}{\sqrt{2j_2+1}} \langle J_3 J_1 M_3 -M_1 | j_2 -m_2\rangle \frac{1}{\sqrt{2j'_3+1}} \langle j_1 j_2 m_1 m_2 | j'_3 -m'_3\rangle =$$

$$= \delta_{j_3 j'_3} \delta_{m_3 m'_3} \frac{1}{2j_3+1} (-)^{\cancel{j_1}+j_2+\cancel{J_1}+\cancel{J_2}} W(j_1 j_2 J_2 J_1; j_3 J_3)$$

$$\sum_{\substack{M_1 M_2 M_3 \\ m_1 m_2}} (-)^{2j_2+M_1+M_2+M_3-m_1-m_2-m_3-m'_3} \langle J_1 J_2 M_1 -M_2 | j_3 -m_3\rangle \langle J_2 J_3 M_2 -M_3 | j_1 -m_1\rangle$$

$$\langle J_3 J_1 M_3 -M_1 | j_2 -m_2\rangle \langle j_1 j_2 m_1 m_2 | j'_3 -m'_3\rangle = \sqrt{\frac{(2j_1+1)(2j_2+1)\cancel{(2j'_3+1)}}{\cancel{(2j_3+1)}}}$$

$$\delta_{j_3 j'_3} \delta_{m_3 m'_3} W(j_1 j_2 J_2 J_1; j_3 J_3) = \sqrt{(2j_1+1)(2j_2+1)}\, \delta_{j_3 j'_3} \delta_{m_3 m'_3}$$

$$W(j_1 J_2 j_2 J_1; J_3 j_3)$$

$$W(J_2 j_1 J_1 j_2; J_3 j_3)$$

$$\langle J_1 J_2 M_1 -M_2 | j_3 -m_3\rangle = \langle J_2 j_3 M_2 -m_3 | J_1 M_1\rangle (-)^{J_2-M_2} \left(\frac{2j_3+1}{2J_1+1}\right)^{\frac{1}{2}}$$

$$= \langle j_3 J_1 m_3 M_1 | J_2 M_2\rangle (-)^{j_3+m_3} \left(\frac{\cancel{2J_1+1}}{2J_2+1}\right)^{\frac{1}{2}} (-)^{J_2-M_2} \left(\frac{2j_3+1}{\cancel{2J_1+1}}\right)^{\frac{1}{2}}$$

$$= (-)^{J_2+j_3-M_2-m_3} \sqrt{\frac{2j_3+1}{2J_2+1}} \langle j_3 J_1 m_3 M_1 | J_2 M_2\rangle$$

$$\frac{\langle \lambda J \| H^{\mathcal{L}} \| \lambda' J' \rangle}{(2J+1)^{\frac{1}{2}}} = \sum_{M'+m=M} \langle J' \mathcal{L} M' m | J M \rangle \langle \lambda J M | H^{\mathcal{L}}_{m} | \lambda' J' M' \rangle$$

φ_{cM} , $c = (\alpha_1, \alpha_2, S, \ell_c, J)$ channel wave function

$$\varphi_{cM} = r_c^{-1} \sum_{\substack{i_1, i_2 \\ \nu, m_c}} \langle I_1 I_2 i_1 i_2 | s \nu \rangle \langle s \ell_c \nu m_c | J M \rangle \psi_{\alpha_1 I_1 i_1}(q_1)\, \psi_{\alpha_2 I_2 i_2}(q_2)\, i^{\ell_c} Y^{\ell_c}_{m_c}(\Omega_c)$$

Note: $\underline{s} = \underline{I_1} + \underline{I_2}$ $\quad \underline{J} = \underline{\ell_c} + \underline{s}$

$$\sum_{\substack{M_1 M_2 M_3 \\ m_1 m_2}} (-)^{\cancel{J_1}+\cancel{J_2}+\cancel{J_3}+M_1+M_2+M_3} \frac{(-)^{\cancel{j_1}-\cancel{j_2}-m_3}}{\cancel{\sqrt{2j_3+1}}} \langle J_1 J_2 M_1 -M_2 | j_3 -m_3 \rangle \frac{(-)^{\cancel{J_2}-\cancel{J_3}-m_1}}{\sqrt{2j_1+1}} \langle J_2 J_3 M_2 -M_3 | j_1 -m_1 \rangle$$

$$\frac{(-)^{\cancel{J_3}-\cancel{J_1}-m_2}}{\sqrt{2j_2+1}} \langle J_3 J_1 M_3 -M_1 | j_2 -m_2 \rangle \frac{(-)^{\cancel{j_1}-\cancel{j_2}-m_3'}}{\cancel{\sqrt{2j_3'+1}}} \langle j_1 j_2 m_1 m_2 | j_3' -m_3' \rangle =$$

$$= \delta_{j_3 j_3'} \delta_{m_3 m_3'} \frac{1}{\cancel{2j_3+1}} (-)^{\cancel{j_1}+\cancel{j_2}+\cancel{J_1}+\cancel{J_2}} W(j_1 j_2 J_2 J_1 ; j_3 J_3)$$

$$\sum_{\substack{M_1 M_2 M_3 \\ m_1 m_2}} (-)^{2j_2+J_3+M_1+M_2+M_3-\cancel{m_3}-m_1-m_2-\cancel{m_3}} \langle J_1 J_2 M_1 -M_2 | j_3 -m_3 \rangle \langle J_2 J_3 M_2 -M_3 | j_1 -m_1 \rangle$$

$$\langle J_3 J_1 M_3 -M_1 | j_2 -m_2 \rangle \langle j_1 j_2 m_1 m_2 | j_3' -m_3' \rangle = \sqrt{(2j_1+1)(2j_2+1)}\, \delta_{j_3 j_3'} \delta_{m_3 m_3'}$$

$W(j_1 j_2 J_2 J_1 ; j_3 J_3)$ $\quad W(j_3 j_2 J_2 J_3 ; j_1 J_1)$

$W(j_2 j_1 J_1 J_2 ; j_3 J_3) = W(j_2 J_1 j_1 J_2 ; J_3 j_3)$

$W(J_1 J_2 j_2 j_1 ; j_3 J_3)$

$$\boxed{\sum_{\substack{M_1 M_2 M_3 \\ m_1 m_2}} \langle j_1 j_2 m_1 m_2 | j_3 m_3 \rangle \langle j_2 J_2 m_2 M_2 | J_3 M_3 \rangle \langle j_3 J_2 m_3 M_2 | J_1 M_1 \rangle \langle j_1 J_3 m_1 M_3 | J_1 m_1 \rangle = \sqrt{(2j_3+1)(2J_3+1)}\, W(j_1 j_2 J_1 J_2 ; j_3 J_3)}$$

$$\langle j_1 j_2 m_1 m_2 | j_3 m_3 \rangle = (-)^{j_1+m_2} \sqrt{\frac{2j_3+1}{2j_1+1}} \langle j_2 j_3 -m_2 m_3 | j_1 m_1 \rangle$$

5. Escribiendo:

$$\langle j_f, m_f | Y_{1m} | j_i, m_i \rangle = (-)^{j_i - j_f + 1} \sqrt{\frac{3}{4\pi}} \sqrt{(2\ell_i+1)(2j_i+1)}\, C(\ell_i, 1, \ell_f ; 00)\, C(1, j_i, j_f ; m, m_i) \times$$
$$\times W(1, \ell_f, j_i, \tfrac{1}{2} ; \ell_i, j_f)$$

se obtiene:

$$\sum_{m, \ell_i} |\langle j_f, m_f | Y_{1m} | j_i, m_i \rangle|^2 = \frac{3}{4\pi} \frac{2j_i+1}{2j_f+1} C^2(\ell_i, 1, \ell_f ; 00) =$$

$$= \frac{3}{8\pi} \frac{\ell_f + \ell_i + 1}{2\ell_i + 1}$$

$$W(1, \ell_f, j_i, \tfrac{1}{2} ; \ell_i, j_f) = (-)^{\ell_i + j_f - \ell_f - j_i} W(1, \ell_i, j_f, \tfrac{1}{2} ; \ell_f, j_i)$$

$$I = \sqrt{\frac{2j_f+1}{2j_i+1}} \sum_m C(1, j_i, j_f ; m_i, m) \langle j_f, m_f | Y_{1m} | j_i, m_i \rangle = \cancel{(-)^{j_i+1-j_f}} \sqrt{\frac{2j_f+1}{2j_i+1}}$$

$$\sum_m C(1, j_i, j_f, m, m_i) \langle j_f, m_f | Y_{1m} | j_i, m_i \rangle = \sqrt{\frac{3}{4\pi}} \sqrt{(2\ell_i+1)(2j_f+1)} \times (-)^{j_i - j_f + 1}$$

$$\times C(\ell_i, 1, \ell_f ; 00)\, W(1, \ell_f, j_i, \tfrac{1}{2} ; \ell_i, j_f).$$

$$\sum_{j_f} I^2 = \frac{3}{4\pi} C^2(\ell_i, 1, \ell_f, 00) = \frac{3}{8\pi} \frac{\ell_f + \ell_i + 1}{2\ell_i + 1}$$

$$W(1, \ell_f, j_i, \tfrac{1}{2} ; \ell_i\, j_f) = W(1, j_i, \ell_f, \tfrac{1}{2} ; j_f, \ell_i)$$

$$|j_i, m_i\rangle = \sum_{m_i'} C(l_i, \tfrac{1}{2}, j_i; m_i', m_i - m_i')\, Y_{l_i m_i'}\, \chi_{m_i - m_i'}$$

$$|j_f, m_f\rangle = \sum_{m_f'} C(l_f, \tfrac{1}{2}, j_f; m_f', m_f - m_f')\, Y_{l_f m_f'}\, \chi_{m_f - m_f'}$$

$$\langle j_f, m_f | Y_{1m} | j_i, m_i\rangle = \sum_{m_i' m_f'} C(l_f, \tfrac{1}{2}, j_f; m_f', m_f - m_f')\, C(l_i, \tfrac{1}{2}, j_i; m_i', m_i - m_i')$$

$$\int Y^*_{l_f m_f'}\, Y_{1m}\, Y_{l_i m_i'}\, d\Omega\; \delta_{(m_i - m_i')(m_f - m_f')}$$

Ya que $\chi^*_{(m)} \cdot \chi_{(m')} = \delta_{mm'}$

$$\int Y^*_{l_f m_f} Y_{1m} Y_{l_i m_i'}\, d\Omega = \sqrt{\frac{3}{4\pi}} \sqrt{\frac{2l_i+1}{2l_f+1}}\, C(l_i, 1, l_f; m_i', m_f' - m_i')\, C(l_i, 1, l_f; 00)\, \delta_{m_f', m_i'+m}$$

$m_i' = m_f' - m$

$$\langle j_f, m_f | Y_{1m} | j_i, m_i\rangle = \sqrt{\frac{3}{4\pi}} \sqrt{\frac{2l_i+1}{2l_f+1}}\, C(l_i, 1, l_f; 00) \sum_{m_i'} C(l_f, \tfrac{1}{2}, j_f; \overset{m_f'}{m_i'+m}, \overset{m_f - m_f'}{m_f - m_i' - m})$$

$$C(l_i, \tfrac{1}{2}, j_i, \overset{m_f'-m}{\cancel{m_i'}}, \overset{m_i - m_f' + m}{\cancel{m_i - m_i'}})\, C(l_i, 1, l_f; \overset{m_f'-m}{\cancel{m_i'}}, m)\, \delta_{(m_i - m_i')(m_f - m_i' - m)}$$

$m_f = m_i + m$

$$C(l_f, \tfrac{1}{2}, j_f; \underset{m_i'+m}{\cancel{m_i'+m}}, \overset{m_f - m_f'}{\cancel{m_f - m_i' - m}}) = (-)^{l_f - m_i' - m} \frac{\sqrt{2j_f+1}}{\sqrt{2}}\, C(l_f, j_f, \tfrac{1}{2}; m_i'+m, -m_f)$$

$$C(l_f, \tfrac{1}{2}, j_f; \overset{m_f'}{\cancel{m_i'+m}}, \overset{m_f - m_f'}{\cancel{m_f - m_i' - m}})\, C(l_i, 1, l_f, m_i', m) = (-)^{l_f - m_i' - m} \sqrt{\frac{2j_f+1}{2}} \quad |||$$

$$C(l_i, 1, l_f; m_i', m)\, C(l_f, j_f, \tfrac{1}{2}; m_i'+m, -m_f) = (-)^{l_f - m_i' - m} \sqrt{\frac{2j_f+1}{2}}$$

a b e α β e d c α+β δ

$$\sum_f \sqrt{(2l_f+1)(2f+1)}\; W(l_i, 1, \tfrac{1}{2}, j_f; l_f, f)\, C(1, j_f, f; m, -m_f)\, C(l_i, f, \tfrac{1}{2}; m_i', m - m_f)$$

Como:

$$\sum_{m_i'} C(l_i, \tfrac{1}{2}, j_i; m_i', m_i - m_i')\, C(l_i, f, \tfrac{1}{2}; m_i'; m - m_f)(-)^{l_f - m_i' - m}$$

$$= \sum_{m_i'} C(l_i, \tfrac{1}{2}, j_i; m_i', m_i - m_i')(-)^{l_f - m_i' - m} (-)^{l_i - m_i'} \sqrt{\frac{2}{2f+1}}\, C(l_i, \tfrac{1}{2}, f; m_i', m_f - m_i' - m)$$

$$= (-)^{l_f - l_i - m} \frac{\sqrt{2}}{\sqrt{2f+1}} \sum_{m_i} C(l_i, \tfrac{1}{2}, j_i, m_i', m_i - m_i')\, C(l_i, \tfrac{1}{2}, f; m_i', m_f - m_i' - m)$$

$$= (-)^{l_f - l_i - m} \frac{\sqrt{2}}{\sqrt{2f+1}}\, \delta_{f j_i}$$

$$\langle j_f, m_f | Y_{1m} | j_i, m_i\rangle = \sqrt{\frac{3}{4\pi}} \sqrt{\frac{2l_i+1}{\cancel{2l_f+1}}}\, C(l_i, 1, l_f; 00)\, \delta_{m_f, m_i+m}\, \cancel{(-)^{l_f - m_i' - m}}$$

$$\sqrt{\frac{2j_f+1}{\cancel{2}}} \sum_f \sqrt{\cancel{(2l_f+1)}\cancel{(2f+1)}}\; W(l_i, 1, \tfrac{1}{2}, j_f; l_f, f)\, C(1, j_f, f; m, -m_f)$$

$$(-)^{l_f - l_i - m} \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2f+1}}}\, \delta_{f j_i} =$$

$l_f - l_i = \pm 1$

$m_i = m - m_f$

$$= \sqrt{\frac{3}{4\pi}} \sqrt{2l_i+1} \sqrt{2j_f+1}\, C(l_i, 1, l_f; 00)\, C(1, j_f, j_i; m, -m_f)\, W(l_i, 1, \tfrac{1}{2}, j_f; l_f, j_i)$$

$$\cancel{(-)^{l_f - l_i - m}} = \circ \quad \delta_{m_f, m_i+m}. \qquad -m+m_f$$

$$C(1, j_f, j_i; m, -m_f) = (-)^{1-m} \sqrt{\frac{2j_i+1}{2j_f+1}}\, C(1, j_i, j_f; m, \overbrace{m_f - m}^{m_i})$$

$$= (-)^{1-m} \sqrt{\frac{3}{4\pi}} \sqrt{2l_i+1} \sqrt{2j_i+1}\; C(l_i, 1, l_f; 00)\, C(1, j_i, j_f; m, m_i)\, W(l_i, 1, \tfrac{1}{2}, j_f, l_f, j_i)$$

$$C(l_i, 1, l_f; 00) = \left\{ \begin{array}{ll} \dfrac{l_i+1}{\sqrt{(2l_i+1)(l_i+1)}} = \sqrt{\dfrac{l_i+1}{2l_i+1}} & l_f = l_i+1 \\ -\dfrac{l_i}{\sqrt{l_i(2l_i+1)}} = -\sqrt{\dfrac{l_i}{2l_i+1}} & l_f = l_i - 1 \end{array} \right\} = (-)^{\frac{1}{2}(l_f - l_i + 1)} \frac{1}{\sqrt{2}} \sqrt{\frac{l_f + l_i + 1}{2l_i+1}}$$

$(l_f - l_i)$

$$W(l_i, 1, \tfrac{1}{2}, j_f; l_f, j_i) = (-)^{l_f + j_i - l_i - j_f}\, W(l_f, 1, \tfrac{1}{2}, j_i; l_i, j_f) =$$

$$= (-)^{l_f - l_i + j_i - j_f}\, W(1, l_f, j_i, \tfrac{1}{2}; l_i, j_f)$$

$$(-)^{1 - \cancel{(m + l_f - l_i)} + j_i - j_f} = (-)^{j_f - j_i - 1}$$

$C(1, j_i, j_f; m, m_i, m_f)$ $\quad m_f = m_i + m.$

-3-

$$C(1, j_i, j_f; m, m_i, m_f) = (-)^{1+j_i-j_f}\, C(j_i, 1, j_f; m_i, m, m_f) \qquad m = \pm 1$$

$j_f - j_i = \pm 1.$

$$C(j_i, 1, j_f, m_i, \underset{\uparrow}{\overset{m_f - m}{m}}, m_f) = \Big\{$$

$$\boxed{\langle j_f, m_f | Y_{1,m} | j_i, m_i \rangle} = (-)^{j_f - j_i - 1} (-)^{\frac{1}{2}(l_f - l_i + 1)} \sqrt{\frac{3}{8\pi}} \sqrt{\cancel{2l_i+1}} \sqrt{2j_i+1} \sqrt{\frac{l_f + l_i + 1}{\cancel{2l_i+1}}}$$

$$C(1, j_i, j_f; m, m_i)\, W(1, l_f, j_i, \tfrac{1}{2}; l_i, j_f)$$

RESULTADO

$$\boxed{= (-)^{\frac{1}{2}(l_f - l_i - 1) + j_f - j_i} \sqrt{\frac{3}{8\pi}} \sqrt{2j_i+1} \sqrt{l_f + l_i + 1}\; C(1, j_i, j_f; m, m_i)\, W(1, l_f, j_i, \tfrac{1}{2}; l_i, j_f)}$$

$$\left(W(1, l_f, j_i, \tfrac{1}{2}; l_i, j_f) = (2l_i+1)^{-\frac{1}{2}} (2j_f+1)^{-\frac{1}{2}}\, U(\quad) \right)$$

$$\sqrt{\frac{2j_i+1}{2j_f+1}}\, C(1, j_i, j_f; m, m_i) = \cancel{(-)^{1-m}}\, \underbrace{C(1, j_f, j_i; m, -m_f)}$$

$$\cancel{\sqrt{\frac{2j_f+1}{2j_i+1}}} \sum_m C(j_i, 1, j_f; m_i, m, m_f)\, C(1, j_i, j_f; m, m_i, m_f) = 1.$$

$$\sum_m \cancel{(-)^{j_i+1-j_f}}\; C(1, j_i, j_f; m_i, m, m_f)$$

$$\sqrt{\frac{2j_f+1}{2j_i+1}} \sum_m C(j_i, 1, j_f; m_i, m, m_f) \langle j_f, m_f | Y_{1,m} | j_i, m_i \rangle$$

Resultado de Jana

$\cancel{1} - l_i + j_i - j_f + \cancel{1} - m_i - m$

$$\sigma_{cap.}(l, m) = \frac{8\pi(l+1)}{l[(2l+1)!!]^2} \frac{k^{2l+1}}{\hbar v} |Q_{lm}|^2$$

$$Q_{lm} = e J_l \langle f | Y_{lm} | i \rangle.$$

$$|Q_{lm}|^2 = e^2 J_l^2 \cdot \frac{3}{8\pi} (2j_i+1)(2l_f + l_i + 1)\, C^2(1, j_i, j_f; m, m_i)\, W^2(1, l_f, j_i, \tfrac{1}{2}; l_i\, j_f)$$

$$\sigma_{cap}(1, m) = \frac{16\pi}{3} \cdot \frac{k^3}{\hbar v} |Q_{1m}|^2$$

$$\sigma_{cap} = \frac{1}{2} \sum_m \sigma(1, m) = e^2 J_l^2 \frac{k^3}{\hbar v} (2j_i+1)(l_f + l_i + 1)\, W^2(1, l_f, j_i \tfrac{1}{2}; l_i\, j_f)$$

$$W^2(1, \underset{\smile}{l_f, j_i} \tfrac{1}{2}; l_i, j_f) = W^2(1, l_i, j_f, \tfrac{1}{2}; l_f\, j_i)$$

$$\sum_{j_i} (2j_i+1)(2l_f+1)\, W^2(1, l_i, j_f, \tfrac{1}{2}; l_f\, j_i) = 1.$$

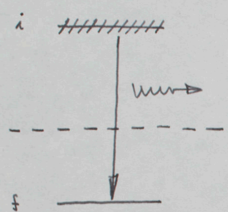
$$\sigma_{cap} = \frac{1}{2} \sum_m \frac{1}{\cancel{(2j_i+1)}} \frac{1}{(2l_f+1)} \sum_{j_i} (2l_f+1)\, \sigma(1, m) =$$

$$= \frac{1}{2j_i+1} \frac{1}{2l_f+1} e^2 J_l^2 \frac{k^3}{\hbar v} (l_f + i + 1).$$

$$k^3 = \left(\frac{\omega}{c}\right)^3 = \left(\frac{\varepsilon_\gamma}{\hbar c}\right)^3$$

$$\frac{1}{\hbar v} = \frac{1}{\hbar \cdot \frac{\hbar k}{M}} = \frac{M}{\hbar^2 k} \qquad \frac{(2j_i+1)}{2(2l_i+1)} \cdot \frac{2}{2(2j_i+1)}$$

CALCULO DEL ELEMENTO DE MATRIZ DIPOLAR EN UNA TRANSICION DEL CONTINUO A UN ESTADO LIGADO USANDO EL MODELO DE CAPAS.



Calcular $\langle f | \mathcal{H} | i \rangle$

en donde

$$\mathcal{H} = \bar{e} r\, Y_{1m}(\theta,\phi)$$

1. $\psi_i = u_i(r)\, | j_i, m_i \rangle$

$$u_i(r) \underset{r\to\infty}{\sim} e^{i \underline{k_i} \cdot \underline{r}} + f(\theta,\phi)\frac{e^{ik_i r}}{r} \qquad \text{(estado de dispersión)}$$

$$| j_i, m_i \rangle = \sum_{m_i'} C(l_i, \tfrac{1}{2}, j_i; m_i', m_i - m_i')\, Y_{l_i, m_i'}\, \chi_{m_i - m_i'}$$

(impulso angular orbital l_i y spin $s=\frac{1}{2}$ se suman para dar j_i y m_i)

(note: $m_i = m_i' + m_s'$)

2. $\psi_f = u_f(r)\, | j_f, m_f \rangle$

$$u_f(r) \underset{r\to\infty}{\sim} \frac{e^{-k_f r}}{r} \qquad \text{(estado ligado)}$$

$$| j_f, m_f \rangle = \sum_{m_f'} C(l_f, \tfrac{1}{2}, j_f; m_f', m_f - m_f')\, Y_{l_f m_f}\, \chi_{m_f - m_f'} \qquad (m_f = m_f' + m_s')$$

3. $$I = \langle f | \mathcal{H} | i \rangle = \bar{e} J \langle j_f, m_f | Y_{1m} | j_i, m_i \rangle$$

en donde:

$$J \equiv \int_0^\infty u_f^*(r)\, u_i(r)\, r^3\, dr$$

4. Como $\chi_m^* \cdot \chi_{m'} = \delta_{mm'}$:

$$\langle j_f, m_f | Y_{1m} | j_i, m_i \rangle = \sum_{m_i' m_f'} C(l_f, \tfrac{1}{2}, j_f; m_f', m_f - m_f')\, C(l_i, \tfrac{1}{2}, j_i; m_i', m_i - m_i')\, \delta_{m_i - m_i', m_f - m_f'} \cdot \int Y^*_{l_f m_f'}\, Y_{1m}\, Y_{l_i m_i'}\, d\Omega$$

Como:

$$\int Y^*_{l_f m_f'}\, Y_{1m}\, Y_{l_i m_i'}\, d\Omega = \sqrt{\frac{3}{4\pi}}\sqrt{\frac{2l_i+1}{2l_f+1}}\, C(l_i, 1, l_f; m_i', m_f' - m_i')\, C(l_i, 1, l_f; 00)\, \delta_{m_f', m_i'+m}$$

$$\langle j_f, m_f | Y_{1m} | j_i, m_i \rangle = \sqrt{\frac{3}{4\pi}}\sqrt{\frac{2l_i+1}{2l_f+1}}\, C(l_i, 1, l_f; 00) \sum_{m_i'} C(l_f, \tfrac{1}{2}, j_f; m_f', m_f - m_f') \times$$

$$\times\, C(l_i, \tfrac{1}{2}, j_i; m_f' - m, m_i - m_f' + m)\, C(l_i, 1, l_f; m_f' - m, m)\, \delta_{m_i + m, m_f}$$

Como:

$$C(l_f, \tfrac{1}{2}, j_f; \underset{m_i'+m}{\underset{\shortparallel}{m_f'}}, m_f - m_f') = (-)^{l_f - m_i' - m}\sqrt{\tfrac{1}{2}(2j_f+1)}\; C(l_f, j_f, \tfrac{1}{2}; m_i' + m, -m_f)$$

resulta:

$$\cancel{C(l_f, \tfrac{1}{2}, j_f; m_f', m_f - m_f')\, C(l_i, 1, l_f; m_i', m) = (-)^{l_f - m_i' - m}\sqrt{\tfrac{1}{2}(2j_f+1)}\; C(l_f, j_f, \tfrac{1}{2}; m_i'+m, -m_f)}$$

$$C(l_f, \tfrac{1}{2}, j_f; m_f', m_f - m_f')\, C(l_i, 1, l_f; \underset{m_i'}{\underset{\shortparallel}{m_f' - m}}, m) = (-)^{l_f - m_i' - m}\sqrt{\tfrac{1}{2}(2j_f+1)} \times$$

$$\times\, C(l_i, 1, l_f; m_i', m)\, C(l_f, j_f, \tfrac{1}{2}; m_i' + m, -m_f) = (-)^{l_f - m_i' - m}\sqrt{\tfrac{1}{2}(2j_f+1)} \cdot$$

$$\sum_f \sqrt{(2l_f+1)(2f+1)}\; W(l_i, 1, \tfrac{1}{2}, j_f; l_f, f)\, C(1, j_f, f; m, -m_f)\, C(l_i, f, \tfrac{1}{2}; m_i', m - m_f)$$

$$= (-)^{j_i - j_f - l_i - m_i' - m}\sqrt{\tfrac{1}{2}(2j_f+1)} \sum_f \sqrt{(2l_f+1)(2f+1)}\; W(1, l_f, j_i, \tfrac{1}{2}; l_i, j_f)$$

$$C(1, j_f, f; m, -m_f)\, C(l_i, f, \tfrac{1}{2}; m_i', m - m_f) =$$

$$= (-)^{j_i - j_f - m} \sqrt{(2j_f+1)(2\ell_f+1)}\; W(1,\ell_f,j_i,\tfrac{1}{2};\ell_i,j_f) \sum_f C(1,j_f,f;m,-m_f) \times$$
$$\times\, C(\ell_i,\tfrac{1}{2},f;m_i',m_f-m_i'-m)$$

ya que:

$$W(\ell_i,1,\tfrac{1}{2},j_f;\ell_f,j_i) = (-)^{\ell_f-\ell_i+j_i-j_f}\, W(1,\ell_f,j_i,\tfrac{1}{2};\ell_i,j_f)$$

y $$C(\ell_i,f,\tfrac{1}{2};m_i',m-m_f) = (-)^{\ell_i-m_i'} \sqrt{\frac{2}{2f+1}}\; C(\ell_i,\tfrac{1}{2},f;m_i',m_f-m_i'-m)$$

Resulta:

$$\langle j_f,m_f|Y_{1m}|j_i,m_i\rangle = \sqrt{\frac{3}{4\pi}}\sqrt{\frac{2\ell_i+1}{2\ell_f+1}}\; C(\ell_i,1,\ell_f;00) \sum_{m_i' f} (-)^{j_i-j_f-m} \sqrt{(2j_f+1)(2\ell_f+1)}\, \times$$
$$W(1,\ell_f,j_i,\tfrac{1}{2};\ell_i,j_f)\; C(\ell_i,\tfrac{1}{2},j_i;\underbrace{m_f-m}_{m_i'},\underbrace{m_i-m_f+m}_{m_i-m_i'})\; C(1,j_f,f;m,-m_f)\,\times$$
$$\times\, C(\ell_i,\tfrac{1}{2},f;m_i',\underbrace{m_f-m_i'-m}_{m_i-m_i'})\,\delta_{m_i+m,m_f} = \sqrt{\frac{3}{4\pi}}\sqrt{(2\ell_i+1)(2j_f+1)}\; C(\ell_i,1,\ell_f;00)$$
$$(-)^{j_i-j_f-m}\; W(1,\ell_f,j_i,\tfrac{1}{2};\ell_i,j_f)\; C(1,j_f,j_i;m,-m_f)$$

ya que:

$$\sum_{m_i'} C(\ell_i,\tfrac{1}{2},j_i;m_i',m_i-m_i')\; C(\ell_i,\tfrac{1}{2},f;m_i',m_i-m_i') = \delta_{j_i f}$$

Como:

$$C(1,j_f,j_i;m,-m_f) = (-)^{1-m}\sqrt{\frac{2j_i+1}{2j_f+1}}\; C(1,j_i,j_f;m,m_i)$$

y: $$C(\ell_i,1,\ell_f;00) = (-)^{\frac{1}{2}(\ell_f-\ell_i+1)}\frac{1}{\sqrt{2}}\sqrt{\frac{\ell_f+\ell_i+1}{2\ell_i+1}} = \begin{cases} \sqrt{\dfrac{\ell_i+1}{2\ell_i+1}}, & \ell_f = \ell_i+1 \\ -\sqrt{\dfrac{\ell_i}{2\ell_i+1}}, & \ell_f = \ell_i-1 \end{cases}$$

se obtiene:

$$\langle j_f,m_f|Y_{1m}|j_i,m_i\rangle = \sqrt{\frac{3}{8\pi}}\,(-)^{\frac{1}{2}(\ell_f-\ell_i-1)+j_f-j_i}\sqrt{2j_i+1}\sqrt{\ell_f+\ell_i+1}\; C(1,j_i,j_f;m,m_i)\, W(1,\ell_f,j_i,\tfrac{1}{2};\ell_i,j_f)$$

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$$W(1,\ell_f,j_i,\tfrac{1}{2};\ell_i,j_f) \simeq W(1,\ell_i,j_f,\tfrac{1}{2};\ell_f,j_i) =$$
$$= W(1,j_f,\ell_i,\tfrac{1}{2};j_i,\ell_f)$$

$\hbar k = mv$

$$\sum_{j_i}(2j_i+1)\, W^2(1,j_f,\ell_i,\tfrac{1}{2};j_i,\ell_f) = \frac{1}{(2\ell_f+1)}$$

$$\sum = \frac{2\ell_i+1}{2\ell_f+1}$$

$$\sigma(\;) = \frac{8\pi}{3}\frac{m}{\hbar^2 k_i}\left(\frac{\omega}{c}\right)^3 J^2 \bar{e}^2 \sum_{\ell_i \ell_f} (\ell_f+\ell_i+1)\; \cancel{\frac{2\ell_i+1}{2\ell_f+1}}$$

$$\ell_f = \ell_i \pm 1$$

$$2\ell_f+1 = 2\ell_i \pm 2+1$$

$$\frac{2\ell_f+1}{2\ell_i+1} = \frac{2\ell_i+1}{2\ell_f+1}$$

$$(2\ell_f+1)^2 = (2\ell_i+1)^2$$

$$\cancel{\ell_f = \ell_i}$$

$$\cancel{2\ell_f+1 = -2\ell_i-1}$$

$$\cancel{2\ell_f+2 = -2\ell_i}$$

$$\cancel{\ell_f+1 = -\ell_i}$$

$$\ell_f = \ell_i - 1$$

$$(2\ell_i-2+1)^2$$

$$(2\ell_i-1)^2$$